2019

BCA

2nd Semester Examination

Mathematical Foundation for Computer Science

Paper – 1203

Full Marks – 70

Time : 3 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer Q. No. *1* and any **six** from the rest taking at last one from each group.

1. Answer any **five** qustions. 2×5

   (a) Apply Descartes rule of signs to determine the nature of the roots of the equation $x^7 - 2x^4 + 2x^3 - 1 = 0$.

   (b) If α, β, γ be the roots of the equation $x^3 + 5x^2 + 1 = 0$. Then find the value of $\sum \frac{1}{\alpha}$.

   (c) *State Taylors mean* value theorem with Lagrange form of remainder.

(d) Verify Rolle's theorem for the function $f(x)=x^2 - 5x + 10$ on $[2,3]$.

(e) Geometrically interprets the Lagrange mean value theorem.

(f) Is $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ an orthogonal matrix ? Verify.

(g) For a square matrix A, show that $A-A^T$ is a skew -symmetric matrix.

(h) State the fundamental theorem of classical algebra.

(i) If X is Poisson variate with parameter μ and $P(X=0) = P(X=1)=k$, prove that $\mu = 1$ and $k=e^{-1}$.

Group – A

(Algebra)

2. (a) Prove that

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3.$$

5

(b) It the roots of the equation $x^3+ax^2+bx+c=0$ are in G.P., then show that $b^3 = a^3c$. 5

3. (a) If the roots of the equation $x^3+px^2+qx+r=0$ are α, β, γ then find the equation whose roots are $\frac{\alpha}{\beta+\gamma}, \frac{\beta}{\gamma+\alpha}, \frac{\gamma}{\alpha+\beta}$. 5

(b) Find the eigen values and eigen vectors of the matrix $\begin{pmatrix} 3 & 0 & 3 \\ 0 & 3 & 0 \\ 3 & 0 & 3 \end{pmatrix}$. 2+3

4. (a) Using Cramer's rule solve the following system of equations $2x-y=3$, $3y-2z=5$, $2z-x=-4$.

(b) Find the adjoin matrix of the following matrix $\begin{pmatrix} 4 & 0 & 1 \\ 1 & 3 & 1 \\ 0 & 7 & 5 \end{pmatrix}$. Also, find the inverse of it.

(5+5)

Group – B

(Calculus)

5. (a) If $f(x,y) = x \sin^{-1}\frac{x}{y} + x\tan^{-1}\frac{y}{x}$ then find the value of $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$. 5

(b) If $y = \cos(m \sin^{-1}x)$, prove that

$(1-x^2)yn+2-(2n+1)xyn+1+(m^2-n^2)\ yn=0$ 5

6. (a) Evaluate any **one** of the integral 5

(i) $\int_0^1 \frac{5x\ \ dx}{(x+2)(x^2+1)}$

(ii) $\int \frac{dx}{(1-x)\sqrt{1-x^2}}$

(b) Evaluate 5

$$\lim_{n\to\alpha}\left[\left(1+\frac{1}{n^2}\right)^{2/n^2}\left(1+\frac{2^2}{n^2}\right)^{4/n^2}\cdots\left(1+\frac{n^2}{n^2}\right)^{2n/n^2}\right]$$

7. (a) Prove that

$$\lim_{n\to\infty}\frac{1}{n}\left\{\sin\frac{t}{n}+\sin\frac{2t}{n}+\cdots+\sin\frac{(n-1)t}{n}\right\}$$

$$=\frac{1-\cos t}{t}$$

(b) Derive the series expansion of the function $f(x)=\log(1+x)$ using Maclaurin's theorem.

(5+5)

8. (a) Explain Skewner and Kwetosis with geometrical concept. 5

(b) Find mean and median of the distribution given by the probability density function $f(x)=kx(1-x), 0\le x\le 1.$ where K is a suitable constant to be calculated. 5

9. (a) Calculate arithmetic mean and standard deviation of the following distribution. 6

Class Interval :	0-9	10-19	20-29	30-39	40-49	50-59
Frequancy :	15	20	25	24	34	12

(b) A random variable X has probability density function $f(x)=\begin{cases}1/4, -2<x<2\\0 \quad \text{elsewhere}\end{cases}$

find (i) P(IxI>1)

(ii) P(2x+5>7) 2+2

10. (a) State Bayes's theorem. Hence, find the probability that the couple has two boys? Given that the couple has 2 children and the older child is boy. Also, given that the probabilities of having a boy or a girl are both 50%.

(b) Find the mean number of heads in the three tosses of a coin. 5+5
