NEW

Part–III 3-Tier

2019

STATISTICS

(Honours)

PAPER—VI

Full Marks : 100

Time : 4 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group—A

(Statistical Inference—II)

[35 Marks]

Answer Q. No. *1* and any *one* from Q. Nos. 2 and 3

1. Answer any *five* questions. 5×5

(a) Let a random variable X follow the $N(0,1)$ distribution, where O is unknown. How will you use the Neyman Pearson *lemma* to find the uniformly most powerful test for $H_0 : \theta = \theta_0$ against $H_1 : \theta > \theta_0$.

(b) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from Bernoilli (p) population. Show that for large n, the asymptotic distribution of $\sin^{-1}\sqrt{x}$

where $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ is $N\left(\sin^{-1}\sqrt{p}, \frac{1}{4n}\right)$.

(c) Write short note on sequential probability ratio test for a simple null hypothesis against a simple alternative hypothesis.

(d) Describe the Mann Whitney Wilcoxon test for testing the identity of two absolutely continuous distributions.

(e) Define unbiased test. Show that any uniformly most powerful test is unbiased.

(f) Given a random sample of size n from $N(\mu, \sigma^2)$ distribution with all parameters unknown, describe the likelihood ratio test procedure for testing $H_0 : \mu = 0$ against $H_1 : \mu > 0$.

(g) Define power, level of significance and size of a non-randomised test. Also state their inter-relationship.

(h) Derive shortest confidence interval for the variance of a normal population with unknown mean.

2. Derive the standard error of the sample standard deviation(s) of a normal sample and hence find the large sample standard error of $l_n s$. 10

3. (a) Find the Most powerful size α test for testing

$H_0 : X \sim N(0, \frac{1}{2}(= \text{variance}))$ against $H_1 : X \sim \text{Cauchy}(0,1)$

based on a single observation S. Also find the power of the test in terms of α. 7

(b) State Neyman Pearson lemma. 3

Group—B

(Theory of Sample Survey)

[35 Marks]

Answer Q. No. 4 and *one* from Q. Nos. 5 and 6

4. Answer any *five* questions. 5×5

(a) Instead of unstratified simple random sampling with replacement (SRSWR) if you resort to stratified

SRSWR procedure with proportional allocation, how much do you gain in precision for estimating the population mean.

(b) In simple random without replacement sampling of n units from the h[th] stratum of size N_h, show that an unbiased estimator of a population proportion (P) of units possessing a certain attribute is given by

$$\hat{p} = \sum_n W_h p_h \text{ and } V(\hat{p}) \doteq \sum_n W_h^2 \frac{1-f_h}{n_h} \frac{N_h p_h(1-p_h)}{N_{h-1}}$$

Also prove that an unbiased estimator of $V(\hat{p})$ is given

by $V(\hat{p}) = \sum_n W_h^2 (1-f_n) \frac{p_h(1-p_h)}{n_h - 1}$ if $n_h \geq 2 \forall h$ where

$W_h = \frac{N_h}{\sum_n N_h}$, $f_n = \frac{n_h}{N_h}$ and ph, P_h are respectively the

sample and population proportion in the h^{th} stratum.

(c) For two stage sampling with simple random without replacement sampling at both the sages, obtain an unbiased estimator of the population total assuming that the first stage units are of equal sizes. Also find an unbiased estimator of the variance of the estimator.

(d) Derive approximate expression for the mean square errors of the ratio and regression estimators of a finite population mean under simple random without replacement sampling.

(e) State some advantages of sample survey over complete census.

(f) Distinguish between

(i) Sampling errors and non-sampling errors

(ii) Linear systematic sampling and circular systematic sampling.

(g) Discuss the situation where double sampling technique is appropriate.

(h) What is single stage cluster sampling ?

How is it different from direct sampling and two stage sampling ?

5. (a) Let c_i be the cost of sampling a unit from the i^{th} stratum and C_0 the overhead cost. Assuming a linear

cost function, find the optimum allocation of the total sample size (n) to the different state. Under what conditions, will this allocation reduce to proportiona allocation ? 5

(b) What are interpenetrating sub samples ? When are they used ? 5

6. (a) A simple random with replacement sample of size 3 is drawn from a population of size N. Show that the probabilities that the sample contains 1, 2 and 3 distinct units are respectively $P_1 = \frac{1}{N_2}$, $P_2 = \frac{3(N-1)}{N^2}$ and $P_3 = \frac{(N-1)(N-2)}{N^2}$. Consider an estimator $\tilde{y}$ which is the unweighted mean over the distinct units in the sample. Show that $\tilde{y}$ is an unbiased estimator of the population mean. 3

(b) Prove that with usual notations, $V_{\text{opt.}} \leq V_{\text{prop.}} \leq V_{\text{rand.}}$ 7

Group—C

(Statistical Quality Control)

[20 Marks]

Answer Q. No. 7 or Q. No. 8 and Q. No. 9

7. Answer any *two* questions. 2×5

(a) Distinguish between

(i) Producer's Risk and Consumer's Risk

(ii) Product control and process control.

(b) How do you calculate the control limits for a *np* chart ?

(c) Describe the single sampling inspection plan for inspection by variables when lower specification limit is known and assuming normal distribution with unknown standard deviation.

(d) Derive the expression for the ASN in double sampling inspection plan.

8. Describe the construction of modified control charts. When are they used ? 10

9. (a) Distinguish between chance and assignable causes of variation. Explain with examples. 5

(b) Describe the construction of Schewart control chart for $\bar{X}$ for detection of lack of control in a continuous flow of manufactured product. 5

[Internal Assessment—10 Marks]
