2019

Part – II

STATISTICS

(Honours)

Paper – III

Full Marks – 90

Time : 4 Hours

Write the answers to Questions of each / Half / Part /Group in separate books wherever necessary.

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

GROUP – A

1. Answer any five questiono : 5×5=25

(a) Show that the sequence of random variables $X_n \xrightarrow{L} X$ where $X_n \sim N(O, \frac{1}{n})$ and X is a random variable having distribution function 5

$$F(x) = \begin{cases} 0 & ; \quad x < 0 \\ 1 & ; \quad x \geq 0 \end{cases}$$

(b) Examine if Weak Law of Large Numbers (WLLN) holds for the sequence of mutually independent random variables $\{x_n\}_P$ where

$$P\left\{X_k = \frac{1}{2k}\right\} = P\left\{X_k = -\frac{1}{2k}\right\} = \frac{1}{2}$$ 5

(c) Derive the mean and variance of χ^2 distribution with n degres of freedom. 5

(d) If random variables x_2 be independently distributed as uniform (0,1), then show that

$Z_1 = \sqrt{-2 \ln x_1} \, Sin \, 2\Pi X_2$ and

$Z_2 = \sqrt{-2 \ln x_1} \, Cos \, 2\Pi X_2$

are independently distributed as $N(0,1)$. 5

(e) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from Rec (0,0). Obtain the probability density functions of the distribution of the longest order statistic $X(n) = \max\{x_1, x_2, \ldots\ldots x_n\}$ and the smallest order statistic

$X_{(1)} = \min\{x_1, x_2, \ldots\ldots\ldots x_n\}$. 5

(f) In drawing a SRSWOR sample of size n from a population of size N, obtain the probabilities that 5

(i) any population unit is drawn at any draw.

(ii) a particular population unit is included in the sample.

(g) State De-Moivers Laplace Limit theorem. Show that Binomial distribution follows the theorem. 2+3

2. Answer any one question :- 10×1=10

(a) Let $X_1, X_2, \ldots\ldots X_n$ be a sample of size n drawn from an exponential distribution with p.d.f $f(x) = \lambda_e^{-\lambda x}$. Derive the sampling distribution of the sample range. 10

(b) (i) State Chebyshev's inequality. 2

(ii) Use the above inequality to show that

$P(T^2 \geq 25) \leq \frac{1}{3}$ where

$T = X_1 + X_2 + \ldots X_{100}$

and $X_1, X_2, \ldots, X_{100}$ are i,i,d uniform $(-0.5, 0.5)$ random variables. 8

GROUP – B

3. Answer any **four** questions : 5×4=20

(a) (i) State Neyman Fisher's factorization Theorem. 2

(ii) Use this theorem to find a sufficient statistic for a when X_1, X_2, X_3 are independent random variables with $X_k (k = 1,2,3)$ have the probability density function 3

$$f_k(x) = \begin{cases} K\theta^{-k\theta x} & ; 0 < x < \infty \\ 0 & 0.w. \end{cases}$$

(b) Describe the exact test procedure for testing the equality of two independent Binomial proportions Ho: $P_1 = p_2$ against all possible alternatives. 5

(c) (i) State and prove the sufficient conditions for consistency of an estimator. 3

(ii) Let $X_1, X_2, ..., X_n$ be i,i,d $B(1, \theta)$ random variobles, $0 < \theta < 1$. Check

whether $T(X_1, X_2, \ldots, X_n) = \dfrac{\sum_{i=1}^{n} x_i + \sqrt{\frac{n}{2}}2}{n + \sqrt{n}}$

is a consistent estimator of θ.

(d) Discuss the test procedure for testing the equality of the variances of two independent normal populations when the means are unknown, Ho! $\sigma_1^2 = \sigma_2^2$ against all possible alternatives. 5

(e) (i) Let X ~ Bin(n,p) where n is known. Show that the only estimable parametric function of þ is a polynomial in þ of degree at most n.

(ii) Let $X_1, X_2, ..., X_n$ be i.i.d random variables with the probability density function

$$f(x/\theta) = \frac{2\theta^2}{X^3}; X \geq 0$$

$$= 0 \text{ O.W}$$

where θ(>0) is unknown. Find the maximun likelihood estimator of θ. 2.5

4. Answer any **one** question : 10×1=10

(a) Derive, stating the regularity conditions, the Gamer Rao lower bound to the variance of an unbiased estimator of an unknown parameter. Also state when equality holds in the lower bound. 8+2

(b) Let $\bar{x} = 9, Sx^2 = 6$ be the sample mean and variance, respectively, based on a random sample of size 3 from N (μ_1, δ^2) . Also let $\bar{y}=7,\ s^2y = 4$ be the sample mean and variance respectively based on a random sample of size 3 from N($\mu_2, 2\sigma_6 2$)where $\mu_1, \mu_2 \varepsilon R$ and $\sigma^2 > O$ are unknown. Find a 95% confidence interval for Given : $P(t_4 \leq 2.78) = 0.975,\ P(t_4 \leq 2.13) = 0.95$ $P(t_5 \leq 2.57) = 0.975,\ P(t_5 \leq 2.01) = 0.95$ where t_n denotes t_ random variable with n degrees of freedom. 10

GROUP – C

5. Answer any **three** questions : 5×3=15

(a) Obtain the characteristic function of multivariate normal distribution. 5

(b) Show that the multiple correlation co-efficient of X_1 on $X_2, X_3, .., X_p$ that is π 1.23... p lies between o and 1. Write the implication of this result. 5

(c) If the random variables $(X_1, X_2, ... X_k)$ follows multinomial distribution with parameters n, p_1, $p_2, ..., p_k$. Obtain the mean vector $\underline{\mu}$ and the dispersion matrix $\sum$ of $\underline{X} = (X_1, X_2 .., X_k)$. 5

(d) What do you mean by partial correlationco-efficient ? Express the partial correlationco-efficient in terms of the elements of the correlation matri. 5

(e) Explain concentration ellipsoid for two variables and extend the same the case of p-variables. 5

6. Answer any **one** question :- 10×1=10

(a) Show that $\underset{\sim}{X}^{b x 1} \sim NP(\underset{\sim}{\mu}, \Sigma)$ iff for any non random vector $\underset{\sim}{l}^{px1}, \underset{\sim}{l} \sim \underset{\sim}{l}^{1} \underset{\sim}{x} \sim N(\underset{\sim}{l}^{l} \underset{\sim}{\mu}, \underset{\sim}{l}^{l} \Sigma \underset{\sim}{l})$ 10

(b) Show that the multiple correlation co-efficient is the maximum value of the correlation co-efficient between variable X_1 and any linear function of $(p-1)$ variables $X_2, X_3, ... X_p$. 10
