NEW

Part–III 3-Tier

2019

PHYSICS

(Honours)

PAPER—VI

Full Marks : 90

Time : 4 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group—A

Answer any *two* questions.

1. (a) For a decimal harmonic oscillation with mass m and angular frequency w, define

$$\hat{a}_{\mp} = \sqrt{\frac{mw}{2\hbar}}\left(\hat{x} \pm i\frac{\hat{p}_x}{mw}\right),$$

where the other terms and symbols are of usual meaning. Show $\hat{a}_+\,\hat{a}_-$ that represent number operations, such that $\hat{N}\varphi_n = n\varphi_n$ when $\hat{N} = \hat{a}_+\,\hat{a}_-$ and n *is the eigen value* in the number state specified by

(Turn Over)

φ_n. Based on the defined operators, find the expectation values of kinetic and potential energy in the ground state of the harmonic oscillator. 6

(b) For the mentioned oscillator, show that the probability of finding the particle within classical turning points is greater than 0.8. 4

(c) Show that $\frac{d}{dt}\langle \hat{p}_x \rangle = -\left\langle \frac{\partial V}{\partial x} \right\rangle$, $\hat{p}_x$ and V denote the momentum operator for x-directional motion and position dependent potential. 3

(d) Show that, for two Hermitian operators $\hat{A}$ and $\hat{B}$, $(\hat{A}\hat{B})^+ = \hat{B}^+\hat{A}^+$. 2

2. (a) What do you mean by Q-value of a nuclear reaction? For a nuclear reaction $X(x, y)Y$ show that,

$$Q = T_y\left(1 + \frac{m_y}{M_Y}\right) - T_x\left(1 - \frac{m_x}{M_X}\right) - \frac{2}{M_Y}\sqrt{m_x m_y T_x T_y}\cos\theta$$

where θ is the scattering angle. 6

(b) Show that the threshold energy for an endoergic reaction is give by

$$E_{\text{th}} = -\frac{(m_y + M_Y)Q}{M_X - Q} \simeq -Q\left(1 - \frac{m_x}{M_X}\right)$$

for $Q \ll m_x$. 4

(c) How does the cross-section for the neutron and proton induced nuclear reactions vary with energy ? Explain. 5

3. (a) Draw the trajectory of a Plank oscillator in a two dimensional phase space. Determine the number of cells in the mentioned phase space for the given oscillator having energy between 0 and ε. Given that the mass of the oscillator is m and its frequency is ν. 1½+2½

(b) What is Gibbs paradox ? Following this paradox, determine the entropy for mixing of two gases considering appropriate partition function for the system. What does the final expression of the entropy imply physically ? 2+4+2

(c) Two classical particles have energy states $E = 0, \varepsilon, 2\varepsilon$ with degeneracies 1, 2, 4 respectively. Find the average energy of the system. 3

4. (a) If $[hkl]$ be the Miller indices of a lattice plane with axial units a, b and c, this show that the separation between two successive planes is given by

$$d = \frac{1}{\sqrt{\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}}}$$ 3

(b) Find the packing fraction of F.C.C structure. 3

(c) Explain paramagnetic Curie-temperature interms of Curie-weiss law. Write down the basic assumptions needed for quantum theory of paramagnetism. What is the significance of effective number of Bohr-magneton ? 2+2+2

(d) Determine the expression of the Clausius-Mossotti Relation in respect of local field effect in the polarization in di-electric material. 3

Group—B

Answer any *five* questions.

5. (a) Consider the potential barrier of the form

$$\upsilon(x) = \upsilon_0 \; ; \quad |x| < \frac{a}{2}$$
$$= 0 \; ; \quad |x| > \frac{a}{2}$$

In the region $|x| < \frac{a}{2}$, the wave function is given by $\varphi(x) = Ae^{-\alpha x}$ with A and α as constants. Calculate $<x>$, $<x^2>$ and show that $\Delta x \sim \frac{1}{\alpha}$. 5

(b) Find the average distance of 1S electron from the nucleus if the wavefunction for 1S orbit of hydrogen atom is

$$\psi_{1,0,0} = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$$

where a_0 is Bohr radius. 3

6. (a) Give a short note on Bohr's postulate of compound nucleus. 5

(b) What is Ghoshal's experiment ? 3

7. (a) What do you mean by 'dead time' in GM counter ? A GM counter has a deadtime of 400 μs. What are the true counting rates when the observed rates are 10 per minute ? 2+2

(b) Discuss the origin of Cosmic rays. 4

8. (a) The result of Kronig Penny model is given by

$$p\frac{\sin\alpha a}{\alpha a}+\cos\alpha a=\cos ka$$

where $p=\frac{mV_0ab}{\hbar}$ and $\alpha^2=\frac{2mE}{\hbar^2}$, other symbols have their usual meanings. Plot the curve of the left hand side as a function of αa and draw the conclusions. 3

(b) Prove that the number of possible states in an energy band of a finite crystal is equal to the number of primitive cells in it. 2

(c) Prove that effective mass can be expressed as $m^*=\frac{\hbar^2}{d^2E/dk^2}$. What is meant by negative effective mass. 2+1

9. (a) What do you mean by Bose-Einstein condensation ? How does it differ from a vapour condensing into the liquid state ? 3

(b) Establish a condition on the number of particles of a system at temperature T at which Bose condensation occurs. Also identify the Bose temperature. 5

10. (a) Consider N-particle ideal gas system in a volume V. Derive the expression of translational partition function. Hence, find the form of translational partition function of a diatomic molecule. 3½+1½

(b) For a system of non-interacting phonons, find expression of average energy.

[Given that $\int_0^\infty \frac{x^3}{e^x - 1} dx = \frac{\pi^4}{15}$] 3

11. (a) Give an account of the essential characteristics o' nuclear fission. How does the phenomenon finc application in a nuclear reactor ? 2+2

(b) What are 'prompt' and 'delayed' neutrons ? Explain the role of delayed neutrons in nuclear reactor. 1+1+2

12. Write down the postulates of Debye's theory of specific heat of metal. Draw the plots of density of frequency modes per unit frequency range in case of Debye-model and Einstein's model. At low temperature write down the expression of total energy. What is Debye-temperature ? Briefly state about the shortcomings of Debye-model.

2+(1½+1½)+1+1+1

Group—C

Answer any *five* questions.

13. The raising $(\hat{L}_+)$ and $(\hat{L}_-)$ lowering operators for orbital angular momentum states change the value of m by one unit ; where $m\hbar$ is the eigen value of the operator $\hat{L}_z$

Then show that,

(i) $\hat{L}_-\hat{L}_+ = \hat{L}^2 - \hat{L}_z^2 - \hbar\hat{L}_z$

(ii) $\hat{L}_+ f_l^m = \hbar\sqrt{l(l+1) - m(m+1)}\, f_l^{m+1}$

where f_l^m is the normalised simultaneous eigen states for the operators $\hat{L}_z$ and $\hat{L}^2$. 4

14. Write down Pauli spin-matrices. The spin state of an electron is an eigenstate of $\frac{\hbar}{2\sqrt{2}}(\sigma_x + \sigma_y)$. What is the probability that the z-component of spin of the electron will be $\frac{1}{2}\hbar$? 1+3

15. A spin $\frac{1}{2}$ particle of mass m with charge $-e$ in an external magnetic field B.

(i) What is the Hamiltonian of the system ?

(ii) If S is the spin angular momentum vector, show that $\frac{d\vec{S}}{dt} = -\frac{e}{m}(\vec{S} \times \vec{B})$ quantum mechanically.

1+3

16. Which of the following reactions are allowed or forbidden and why ?

(i) $n \to p + e^-$ (ii) $n \to \pi^+ + e^- + \bar{\nu}_e$

(iii) $\pi^- + p \to \pi^0 + n$ (iv) $\pi^+ \to \mu^+ + \nu_\mu$ 4

17. (a) Calculate the angle between [111] and $[\bar{1}\bar{1}1]$ directions in a cubic crystal.

(b) Consider a set of lattice planes having inerseparation 1.95 Å. If we use X-rays of wavelength λ = 1.542Å, find all possible Bragg angles for reflection from those planes. 2+2

18. (a) An oscillator consists of a weight of 1 kg at the end of a light rod of length 1 m. If the amplitude of oscillation is 0.1 m. Calculate the approximate value of the quantum number. 2

(b) Show that the eigen functions corresponding to distinct eigen values of Hamiltonian are orthogonal. 2

19. Silver (fcc) has an atomic radius of 1.4Å. Assuming silver to be monovalent metal, compute the values of Fermi energy, Fermi temperature and the Fermi velocity.

Given $\hbar = 1.05 \times 10^{-34}$ J-s ; $m_{el} = 9.1 \times 10^{-31}$ kg

$k_B = 1.38 \times 10^{-23}$ J K^{-1} 4

20. (a) What is Meissner effect ?

(b) A superconducting material has a critical temperature 4k in zero magnetic field, and at 0K, the critical field is 0.03 Tesla. Find the critical field at 3K. 2+2

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