2019

Part – II
PHYSICS
(Honours)

Paper – III

Full Marks – 90

Time : 4 Hours

The figures in the right hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary.

(NEW SYLLABUS)

GROUP – A

1. Answer any two questions : 15 x 2

(a) Find the eigen values and eigen vetors of the matrix :

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{pmatrix}$$ 4

(b) prove : $H_{n+1}(x) = 2xH_n(x) - 2nH_{n-1}(x)$ 2

(c) expand the following function in Fourier series :

$f(x) = x^2$ for $-\pi < x < \pi$. 3

Hence show that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$. 2

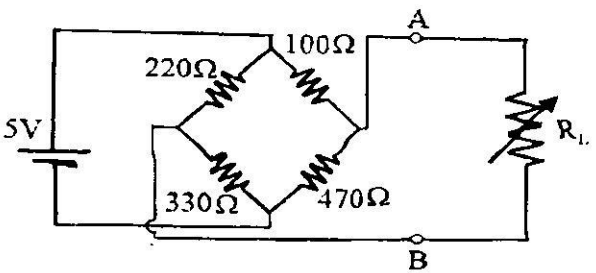
d) Which of the matrices given below is / are Hermitian and which is / are unitary ?

2 + 2

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

2. a) State and prove super position principle. 1+2

b) In the following circuit find the value of R_L for which the circuit on the left of the terminals AB will deliver maximum power to R_L. 2



c) In a parallel LCR circuit an inductance L in series with a resistance R is connected in parallel with a capacitor of capacitance C. An A.C. signal is applied to this parallel LCR combination. Find the parallel resonant frequency and dynamic resistance. 3+2

what is the significance of dynamic resistance for R = 0 ? 1

d) find the expression of self -inductance of a uniform solenoid of finite length L. 4

3. a) Write down Maxwell's equations in free space. Obtain the differential wave equations for electric and magnetic field intensities from them. 2+2

b) Show that for a plane electromagnetic wave in free space the propagation vector, the electric field vector and magnetic field vector are mutually perpendicular. 4

c) Write the expression of Poynting vector $\vec{S}$. Show that 1+3

$$\langle \vec{S} \rangle = \frac{1}{2} \vec{E}_0 \times \vec{H}_0$$

d) An electromagnetic wave , travelling in free space, is incident obliquely and reflected at the plane surface of an isotropic dielectric. If the electric field vector of the wave is parallel to the reflecting surface , write the Fresnel equation for reflection. Comment on the relation between the phases of the incident and reflected wave. 2+1

4. a) Write down the expression of the change in wavelength of a photon undergoing Compton scattering by an angle φ. 2

b) What is Compton wavelength ? Show that the value of compton wavelength is given by :

$$\lambda_c = 0.02424\,\text{Å} \qquad 2$$

c) Nuclei of a radioactive element spend 10^{-1} s in the excited state before coming to the stable ground state by emitting gamma photon. What will be the spread of the emitted gamma ray frequency ? 2

d) Calculate the probability currenct density for the wave: $\psi(x) = u(x)\exp(-i\varphi(x))$, where u and φ are real. 3

e) Show for relativistic case that group velocity represents particle velocity. 2

f) Calculate the de Broglie wavelength of an electon of energy 1 MeV. Why nonrelativistic treatment cannot be applicable in this case? 2+2

5. a) Let $\varphi = a_{jk} A^f A^k$, where a_{jk} may or may not be symmetric. Show that you can always find a set of symmetric b_{jk} so that $\varphi = b_{jk} A^f A^k$. 3

b) Determine the metric tensor in cylindrical coordinates. 2

c) Evaluate the integral $\int_{1+i}^{2+4i} z^2 dz$ along a parabola $x = t$, $y = t^2$, where $1 \le t \le 2$. 3

GROUP – B

Answer any **Five** questions : 8 x5

6. a) if $f(z) = \varphi + i\varphi$ is analytic and $\varphi = x^2 + 4x - y^2 + 2y$ then find φ. 3

b) Using residue theorem evaluate : 3

$$I = \int_{-\infty}^{+\infty} \frac{dx}{1+x^2}$$

c) Verify wheather the matrix

$$A = \begin{pmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{pmatrix}$$ can perform vector rotation operation. 2

7. a) Define binomial distribution stating the condition of its applicability. 2+1

b) Three unbiased coin are tossed. Find the probabilities of (i) 0 head , 2 heads, 3 heads: (ii) more than one head: (iii) at least one head. 3+1+1

8. a) Analyze the A.C balance of an Anderson bridge with equal resistance in the four arms to obtain a suitable expression for the inductance. 4

b) In an Anderson bridge each arm has 100 ohm resistance and the capacitor has a capacitance of $1\mu F$. How small inductance can you measure with the bridge ? 2

c) Find the critical damping resistance of a series LCR circuit containing inductance and capacitance 100 mH and $0.1\mu F$ respectively. 2

9. a) For a wave group the relation between angular frequency ω and propagation constant k is : $\omega^2 = \alpha k + \beta k^3$. Find the value of propagation constant for which phase velocity is equal to group velocity. 3

b) A particle is moving in a two dimensional infinite potential we defined as:

$V(x,y)=0$ for $0 < x < a$ and $0 < y < a$

$V(x,y) = \infty$ otherwise.

Keeping analogy with one dimensional potential well give the expressions of the energy eigen values and eighn functions (need not to be normalized) of the particle. 1+2

c) The wave function of a quantum mechanical particle is given by:

$$\psi(x)\frac{3}{5}\varphi_1(x)+\frac{4}{5}\varphi_2(x).$$

Where $\varphi_1(x)$ and $\varphi_2(x)$ are the eigen functions with energy eigen values –1eV and – 2eV. Find the energy of the particle in the state $\psi(x)$. 2

10. a) State Einstein's assumption in explaning photoelectric equation. 1+3

b) Calculate the rest mass, relativistic mass and momentum of a photon of energy 5eV. 2

c) Express the density of a rod of uniform cross section, moving with velocity v parallel to its length, in terms of its density at rest. 2

11. a) Give Lorentz dispersion equation stating the meaning of each symbol used. 2

b) About 10.05% 0f incident sunlight is lost at 694.3 nm wavelength due to Rayleigh scattering in the atmosphere.Find the percentage of sunlight lost due to scattering in the atmosphere at the wavelength at the wavelength 1060 nm. 3

c) A plane electromagnetic wave propagating in air with $\vec{E} = \left(8\hat{i} + 6\hat{j} + \hat{k}\right)e^{i(-3x+4y-\omega t)}$ is incident on the plane surface of a perfectly conducting slab, the plane surface being in the YZ pane and at x = 0. What will be the electric field vector of the reflected wave? 3

12. a) Starting from Faraday's laws show that for time varying fields $\vec{E} = -\vec{\nabla}\varphi - \frac{\partial \vec{A}}{\partial t}$, where the symbols have their usual meaning. 4

b) Modify Ampere's circuital law for time varying fields and derive Maxwell's fourth equation of electromagnetic induction. What do you mean by displacement current? 3+1

Group – C

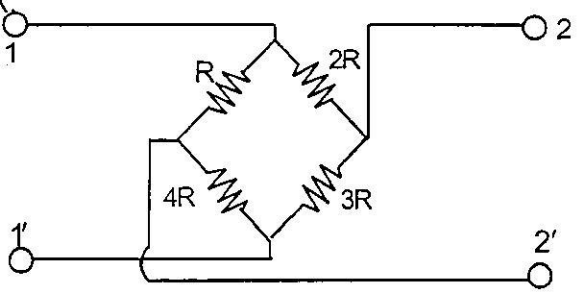
Answer any **five** questions : 4 x5

13. Obtain the eigen values and eigen vectors of 4

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

14. A capacitor C is charged through a resistance R by a battery of emf e. Find the expressions of energy gained by the capacitor and that lost by the battery. Why they are not equal. 4

15. Convert the following network to a T-equivalent network 4



16. a) Solve time independent Schrodinger equation to obtain energy eigen values and normalized eigen functions of a particle in a one dimensional *infinite square well potential* between $x = 0$ and $x = a$. 2

b) A particle freely moving in a 1D infinite square well potential of length a. Find the position(s) where the probability of finding the particle at 1st and 2nd excited states is maximum. 2

17. Intensity of sunlight reaching the earth's surface is about $1300\,W\,.\,m^{-2}$. Calculate the peak values of the electric and magnetic fields of the incoming sunlight. 4

18. State and explain Poynting theorem. Calculate the average value of Poynting vector for a plane wave travelling in vacuum having an electric field amplitude $E_0 = 50\mu V / m$. 2+2

19. a) A meter stick is held in a system at 45° angle with the direction of motion of the system. The system moves with a velocity 0.8c. What will be the length of the meter stickin the laboratory frame? 2

b) Photons of wavelengths 80 nm and 110 nm, falling on a metal surface produce photoelectrons of maximum kinetic energies $11.390\,eV$ and $7.15\,eV$ respectively. Estimate the value of Planck constant. 2

20. Starting from Lorentz transformation equations,derive Einstein's velocity addition theorem. 4
