NEW

Part–III 3-Tier

2019

MATHEMATICS

(Honours)

PAPER—VIII

Full Marks : 60

Time : 4 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group—A

(Numerical Analysis)

[Marks : 25]

1. Answer any *two* questions : 2×8

(a) Describe the iteration process to find the real root of a function $f(x) = 0$ in $[a, b]$ and state the condition for which the iteration process converges with desire degree of accuracy. Find the condition of convergence of the iteration process. Show that the order of convergence of iteration process is linear.

(3+1)+2+2

(Turn Over)

(b) (i) What do you mean by 'round-off errors' in numerical data ? Show how these errors are propagated in a difference table ? 4

(ii) Define k-th order forward difference of a function $f(x)$. Prove that the nth order forward difference of a polynomial of degree n is constant. 1+3

(c) (i) Describe the iteration process to find the solution of a system of non-homogeneous equation $Ax = B$ where $A = [a_{ij}]_{n\times n}, X = [x_1, x_2, \dots x_n]$ $B = [b_1, b_2, \dots, b_n]$ and state its condition of convergence. What modification will be done if this condition is not satisfied ? 3+2

(ii) If $f(x)$ is a function such that $f(0) + f(b) = -107$, $f(1) + f(5) = -36$ and $f(2) + f(4) = -3$, find f(3). ?

2. Answer any *three* questions : 3×3

(a) State general error formula for the functional relation $u = f(x_1, x_2, \dots x_n)$. Find the relative error of S where

$$S = \frac{a^2\sqrt{b}}{c^3} \quad \text{and} \quad a = 6.54 \pm 0.01$$

$$b = 48.64 \pm 0.02 \quad \text{and} \quad c = 13.5 \pm 0.03$$ 1+2

(b) Write down the iterative formula of modified Euler method and state why it is better than Eular method. 2+1

(c) Describe pivoting process to find the solution of non-homogeneous equation in Gauss's Elimination method. 3

(d) Define 'degree of precision' of a numerical integration formula. What are it for Tapez-oidal rule and Simpson's 1/3 rule ? 2+1

(e) What is the principle for the numerical differentiation ? Deduce the differentiation formulae for computing first and second order derivative of a fluction $f(x)$ at the first interpolating point. 1+2

Group—B

(Real Analysis—III)

[Marks : 25]

3. Answer any *one* question : 1×15

(a) (i) Let $D \subset \mathbb{R}$ and for each $n \in N, f_n : D \to \mathbb{R}$ is continuous on D. If each f_n be continuous on D then show that the uniform convergence of the sequence $\{f_n\}$ on D is a sufficient but not a necessary condition for continuity of the limit function on D. 3+2

(ii) Let $\sum a_n x^n$ be a power series with radius of convergence $R(>0)$. Let $f(x)$ be sum of the power series on $(-R, R)$. Then prove that $f(x)$ is continuous on $(-R, R)$. 5

(iii) If $f(x)$ be the sum of the series $e^{-x}+2e^{-2x}+3e^{-3x}+\ldots, x>0$. Show that $f(x)$ is continuous for all $x>0$.

Evaluate $\displaystyle\int_{\log_e^2}^{\log_e^3} f(x)dx$. 5

(b) (i) State and prove Cauchy's criteria for uniform convergence of the sequence of functions. 1+5

(ii) For the series $\displaystyle\sum_{n=1}^{\infty} f_n(x)$ where $f_n(x)=n^2xe^{-n^2x^2}$ $-(n-1)^2xe^{-(n-1)^2x^2}, x\in[0,1]$. Show that $\displaystyle\sum_1^{\infty}\int_0^1 f_n(x)dx \neq \int_0^1\left(\sum_1^{\infty} f_n(x)\right)dx$. Is the series $\displaystyle\sum_1^{\infty} f_n(x)$ uniformly convergent on $[0, 1]$? 4+1

(iii) Show that the sequence of functions $\{f_n(x)\}$ where $f_n(x) = \frac{nx}{1+n^2x^2}, \forall x \in \mathbb{R}$, is not uniformly convergent in any interval $[a, b]$ which contains 0. 4

4. Answer any *one* question : 1×8

(a) (i) Let $D \subset \mathbb{R}$ and let $\{f_n\}$ be a sequence of functions pointwise convegent to $f(x)$. Let $M_n = \sup\limits_{x \in D} |f_n(x) - f(x)|$. Then show that $\{f_n\}$ is uniformly convergent on D to $f(x)$ if and only if $\lim\limits_{n\to\infty} M_n = 0$. 4

(ii) Assuming that the sum of the power series $x - \frac{x^2}{2} + \frac{x^3}{3} \ldots$ on its interval of convergence is $\log_e(1+x)$ deduce that

$$\frac{1}{1\cdot 2} - \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} - \ldots = 2\log 2 - 1$$ 4

(b) (i) Let $f : [-\pi, \pi] \to \mathbb{R}$ be continuous except for at most a finite number of jumps and is periodic of period 2π, then prove that

$$\frac{a_0^2}{2} + \sum_{k=1}^{n} (a_k^2 + b_k^2) \le \frac{1}{\pi} \int_{-\pi}^{\pi} f^2(x)dx$$

where a_k and b_k are the Fourier co-efficients of $f(x)$ defined by

$$a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos nt\,dt, n \geq 0$$

$$b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\sin nt\,dt, \text{for } n \geq 1$$ 4

(ii) Define a uniform continuity of a function on interval $[a, b]$. Show $f(x) = \frac{1}{x}, x \neq 0$ is not uniformly continuous in $(0,1)$. 1+3

5. Answer any *one* question : 1×2

(a) Using Cauchy's principle prove that $\lim_{x\to 0} \cos\frac{1}{x}$ does not exist. 2

(b) Find $\overline{\lim}\, u_n$ and $\underline{\lim}\, u_n$ where $u_n = n + \frac{(-1)^n}{n}$. 2

Group—C

(Linear Algebra)

[Marks : 10]

6. Answer any *one* question : 1×8

(a) (i) A linear transformation $L: V \to W$ has an inverse if and only if it is bijective.

(ii) Let A, the matrix representation of the linear transformation $L: P_1(t) \to P_2(t)$ with respect to the basis $\{1, -t, t\}$ and $\{1, t, 1+t+t^2\}$ be given by

$$A = \begin{bmatrix} 2 & -1 \\ -1 & -1 \\ 1 & 1 \end{bmatrix}.$$ Determine L. 2+2+4

(b) (i) Let V and W be vector spaces over a field F and V is finite dimensional. If $T: V \to W$ be a linear mapping then show that

$$\dim \ker T + \dim \operatorname{Im} T = \dim V.$$ 5

(ii) Find a linear mapping $T: \mathbb{R}^3 \to \mathbb{R}^3$ such that Ker(T) is the subspace $u = \{(x, y, z) \in \mathbb{R}^3 : x - y - z = 0\}$ of $\mathbb{R}^3$. 3

7. Answer any *one* question : 1×2

(a) A linear mapping $T: R^3 \to R^3$ is defined by $T(x, y, z) = (2x + z, x + y + z, -3x - 2z)$, $(x, y, z) \in R^3$. Show that $T^{-1} = T$. 2+1

(b) Let V and W be vector spaces over a field F. If $T : V \to W$ be a linear mapping then prove that $\operatorname{Im} T$ is a subspace of W. 2

2019

Part–III 3-Tier

MATHEMATICS

PAPER—VIII (SET—I)

(Honours)

(PRACTICAL)

Full Marks : 30

Time : 2 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group—D

Problem — 24 + Practical Note Book & Viva-Voce — 6

Answer any *two* questions.

The questions must be allotted by lottery.

Program must be written either in FORTRAN language or in C language.

1. Write a program to find the sum of the following series for $x = \frac{\pi}{6}, \frac{\pi}{3}$.

$$\sum_{i=0}^{\infty}(-1)^i \frac{x^{2i+1}}{(2i+1)!}$$

2. Write a program to compute ${}^nC_r$ using a function by which ${}^xP_3$ can be calculated. Then demonstrate your program using ${}^7P_3$ and ${}^7C_4$.

3. Write a program to find the L.C.M. and GCD between two integers. Demonstrate your program for the integers 64, 96.

4. Write a program to find the sum of all digits in a number.

5. Write a program to find the trace of a matrix which is the transpose of the matrix $A_{m \times n}$ given by Keyboard.

6. Write a program to find the transpose of a matrix which is obtained by addition of two given other matrices.

7. Write a program to check the compatibility of matrix multiplication for a given matrix $A_{m \times n}$ and then compute $A^2_{m \times n}$.

8. Write a program to check whether a number is palindrome or not. Demonstrate your program for the string '7345437' and 'March'.

9. Write a program to search word in a text without using library function. Demonstrate your program for the word 'student' in a text 'Rahim is a good student in a class'.

10. Write a program to change the case of a string from lower case to upper case and vice-versa. Demonstrate your program for the strings 'madam' and 'SIR'.

11. Write a program to count the number of words in a sentence. Demonstrate your program for 'Always do what is right. It will gratify half of mankind and astound the other'.

12. Write a program to write a name of a person in short form. For example, 'Bimal Kumar Samanta' can be written as 'Samanta B. K.'.

13. Write a program to find a root of the equation $x^3 - 2x^2 + x = 3$ by Regula-falsi method, correct up to 5 decimal places.

14. Write a program to evaluate $\int_1^2 x^3 \sin(2x)dx$ by Simpson's $\frac{1}{3}$rd rule taking 100 subintervals.

15. Write a program to solve the equation $dy/dx = 4y^2 - 2x^2 + 5$, $y(0) = 1$ by Fourth order Runge-Kutta method for $x = 0.3$.

16. Write a program to find a real root of the equation $x^3 + 5x - 2 = 0$ by fixed point Iteration method correct upto 5 decimal places.

17. Write a program to find the second and third central moments of a set of 8 numbers. Demonstrate your program for the sample 8, 9, –8, 5, 6, 12, 34, 76.

18. Write a program to find the skewness and kurtosis of a set of 8 numbers. Demonstrate your program for the sample 8, 9, –8, 5, 6, 12, 34, 76.

19. Write a program to prepare a frequency table for a distribution. Demonstrate this for the sample : 12, 34, 56, 5, 10, 34, 60, 40, 21, 25, 20, 15, 5, 34, 55, 60, 12, 5, 10, 15, 60, 21 and 60.

20. Write a program to compute the correlation coefficients for a set of points (x_i, y_i), $i = 1, 2, 3, \ldots, n$. Demonstrate this taking the sample as $x_i = 1, 2, 3, 4, 5, 6, 7, 8$ and $y_i = 1, 4, 9, 16, 25, 36, 49, 64$.

———

2019

Part–III 3-Tier

MATHEMATICS

PAPER—VIII (SET—II)

(Honours)

(PRACTICAL)

Full Marks : 30

Time : 2 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group—D

Problem — 24 + Practical Note Book & Viva-Voce — 6

Answer any *two* questions.

The questions must be allotted by lottery.

Program must be written either in FORTRAN language or in C language.

1. Write a program to find the perimeter and area of

$$(x - 2)^2 + y^2 = 100.$$

2. Write a program to find sin 30° from their infinite series correct up to five decimal places.

3. Write a program to determine the nature and roots of the quadratic equation

$$5x^2 + 7x + 1 = 0.$$

4. Write a program to find the difference between the maximum and next maximum among n inputs.

5. Write a program to check either two integers are co-prime or not.

6. Write a program to reverse the digits of an integer.

7. Write a program to find the values of ${}^{n}P_{r}$ and ${}^{n}C_{r}$ for two positive integers n and r.

8. Write a program to arrance in ascending ordered of a list of n numbers.

9. Write a program to find the product of two conformable matrices.

10. Write a program to add a square matrix and its transpose.

11. Write a program which takes a set of full names as inputs and rewrite the name with surname first followed by initials of first and middle name.

12. Write a program which takes sentence as an input string from the key broad and converts its uppercase characters to lowercase and vice versa.

13. Write a program to find the number of occurrences of a letter 'a' in a given string "Student stays focused on the task at hand".

14. Write a program to find the value of a function at a given point by Newton forward interpolation technique. Demonstrate your program for the following information :

x	1	2.2	3.4	4.6	5.8	7	8.2	9.4	10.6	11.8
y	Q	1.7	4.1	7.0	10.2	13.6	17.2	21.1	25.0	29.1

15. Write a program to find a real root of an equation by Iteration method. Demonstrate your program for the equation $xe^x - 1 = 0$ in the range $[0, 1]$.

16. Write a program to evaluate the integral $\int_0^{\frac{\pi}{2}} x^2\sqrt{\cos x}dx$ numerically by Simpson $\frac{1}{3}$'s rule.

17. Write a program to solve a system of linear equations by Gauss-Siedal iteration method. Demonstrate your program for the following system of linear equations

$$\begin{aligned} 3x + 2y - 9z &= -65 \\ -9x - 5y + 2z &= 16 \\ 6x + 7y + 3z &= 5 \end{aligned}$$

18. Write a program to find solution of a differential equation by Runge-Kutta method. If $\frac{dy}{dx} = \tan y$, $y(1) = 1$, determine $y(1.4)$ using the program.

19. Write a program to determine the mean and standard deviation of a set of input data. Demonstrate your program for the following input data set : {13.2, 47.15, 56.0, 23.2, 77.15, 66.9, 27.15, 51.0, 13.2, 85.5, 46.9}.

20. Write a program to fit a parabola through an input set of bivariate data. Demonstrate your program for the following input data set : $\{(n, n^2 - 5n), n = 1, 2, 3, \ldots, 10\}$.

———