NEW

Part–III 3-Tier

2019

MATHEMATICS

(Honours)

PAPER—VII

Full Marks : 90

Time : 4 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group—A

(Elements of Computer Science)

[Marks : 30]

1. Answer any *two* questions : 2×8

(a) (i) Transform the following DNF into CNF

$x'yz' + x'y'z + xyz + xy'z' + x'y'z'$ 4

(ii) Define full adder and draw a logic circuit of it using NAND gates only. 4

(Turn Over)

(b) (i) State and prove De Morgan's laws and verify them using truth tables. 4

(ii) Write a program in FORTRAN-77 or in C, that inputs an integer and check whether it is palindrome. 4

(c) (i) What are the features of an algorithm ? Write an algorithm to generate first 40 numbers of Fibonacci sequence. 2+3

(ii) Design a two input X-OR gate using only NAND gates. 3

2. Answer any *two* questions : 2×4

(a) The value of e can be approximated by the infinite

series : $e = \sum_{n=0}^{\infty} \frac{1}{n!}$, where $0! = 1$.

Write a program in C or FORTRAN-77 to compute the value of e by direct summation of successive terms neglecting the term whose value is less than 10^{-7}. 4

(b) Explain with example subroutine subprogram in FORTRAN-77. 4

Or,

Explain with example the function in C.

(c) What do you mean by subscripted variable ? How it is implemented in FORTRAN-77 Or, C ? 2+2

3. Answer any *two* questions : 2×3

(a) Write an algorithm to arrange 100 real numbers in ascending order. 3

(b) Why are binary numbers used in computer design instead of decimal numbers ? Convert the decimal number 53:375 to its binary equivalent. 3

(c) Explain user defined function in C. How does it differ from main function ? 2+1

Group—B

(Mathematical Theory of Probability)

[Marks : 35]

4. Answer any *one* question : 1×15

(a) (i) A problem in mathematics is given to $(n-1)$ students whose chances of solving it are respectively $\frac{1}{2}, \frac{1}{3}, \ldots \frac{1}{n}$. What is the probability that the problem will be solved ? 4

(ii) The ray of light is sent in a random direction towards x-axis from the point $Q(0,1)$ on y-axis and the ray meets x-axis at a point P. Find the distribution of the abscissae of P. 5

(iii) Prove that for normal (u, σ) distribution, $u_{2r+1} = 0$ and $u_{2r} = 1.3.5\ldots(2r-1)\sigma^{2r}$, r being positive integer. 6

(b) (i) Define joint distribution $F(x, y)$. Prove that a necessary and sufficient condition for two random variables X and Y to be independent is that $F(x, y)$ can be expressed as 2+3

$$F(x,y) = f(x)\cdot\phi(y)$$

(ii) If χ_1^2, χ_2^2 are independent variates having chi-square distributions with m, n degrees of freedom respectively, then prove that $\dfrac{n\chi_1^2}{m\chi_2^2}$ is a variate. 5

(iii) If the probability density function of a random variable X is given by $f(x) = ce^{-(x^2+2x+3)}, -\infty < x < \infty$ find the value of c, the expectation and variance of the distribution. 5

5. Answer any *two* questions : 2×8

(a) (i) If X is standard normal variate then prove that

$Y = \frac{1}{2} \times X^2$ is $\gamma\left(\frac{1}{2}\right)$ variate. 4

(ii) A random point (X, Y) is uniformly distributed over a circular region $x^2 + y^2 < a^2$. Find the marginal distribution of X and Y and the conditional distribution of Y assuming $X = k$, where $|x| < a$. 4

(b) (i) By the method of characteristic functions show that a χ^2-variate with n degrees of freedom is asymptotically normal $(n, \sqrt{2n})$ variate. 4

(ii) If the mutually independent random variables X_1, $X_2, X_3, \ldots, X_n$ all have the same distribution and their sum $X_1 + X_2 + \ldots + X_n$ is normally distributed then show that each of them is normally distributed. 4

(c) (i) Show that variance of t-distribution with n degrees of freedom exists for $n > 2$ and hence obtain its value. 3+1

(ii) A continuous distribution has probability density function $f(x) = ae^{-ax}, 0 < x < \infty, a > 0$. Calculate moment generating function and hence obtain α_k. 4

6. Answer any *one* question : 1×4

(a) Let X and Y are independent, $\gamma(l)$ and $\gamma(m)$ variates respectively. Prove that $X + Y$ and $X/(X+Y)$ are independent, $\gamma(l + m)$ and $\beta(l, m)$ variates respectively. 4

(b) State and prove Chebyshev's inequality. 4

Group—C

(Mathematical Statistics)

[Marks : 25]

7. Answer any *one* question : 1×15

(a) (i) Explain the term random sample and write the difference between sampling distribution and distribution of sample. 3

(ii) Find the sampling distribution of the statics

$t = \dfrac{\sqrt{n}(\bar{x} - m)}{s}$, where $\bar{x}$ is the sample mean and

$(n-1)s^2 = \sum_{i=1}^{n}(x_i - \bar{x})^2$. ($x_1, x_2, \ldots, x_n$ represent a sample of size n from a normal population of mean m and variance σ^2.) 7

(iii) Show that regression coefficients independent of change of origin but depends on change of scales of variable. 5

(b) (i) Apply the method of likelihood ratio testing to develope a method of testing the hypothens H_0 : $p = p_0$ for binomial (n, p) population, when n is known and large. 6

(ii) Define likelihood function. Find the maximum likelihood estimate of the parameter λ of a continuous population having the density function

$f(x) = \lambda x^{\lambda - 1}, 0 < x < 1$ where $\lambda > 0$. 5

(iii) In a group of 10 students, a dull students secured 25 marks below the average marks of the other students. Prove that the standard deviation of marks of all students is at least 7.5. If this standard deviation is actually 12, find the standard deviation when the dull student is left out.

8. Answer any *one* question : 1×8

(a) (i) The heights in inches of 7 students of a college, chosen at random were as follows :

65.3, 68.4, 68.2, 64.2, 62.3, 64.5, 61.5

Compute 95% confidence intervals for the mean and standard deviation of the population of height of students of the college, assuming it to be normal.

Given $P(t > 2.447) = 0.025$

$P(\chi^2 > 1.218) = 0.975$

$P(\chi^2 > 14.626) = 0.025$

for 6 degree of freedom. 5

(ii) Distinguish between a population and a sample. 3

(b) If r be the sample correlation co-efficient of a bivariate sample $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$, then prove that $-1 \le r \le 1$, also prove that the sample correlation co-efficient is independent of the unit of measurement and of the choice of origin. Given signification for $r = 0$. 4+3+1

9. Answer any *one* question : 1×2

(a) State Neymann Pearson theorem on the best critical region of a test of hypothesis. 2

(b) Show that sample mean is consistent estimates of population mean. 2
