NEW

Part–III 3-Tier

2019

MATHEMATICS

(Honours)

PAPER—VI

Full Marks : 90

Time : 4 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group—A

(Rigid Dynamics)

[Marks : 30]

1. Answer any *three* questions : 3×8

   (a) If A and B be the two moments of inertia of a plane lamina about rectangular axes of OX and OY in the plane of lamina and F be the product of inertia with regard to these axes, find the moments of inertia A'

(Turn Over)

and B' and the product of inertia of F' of the lamna about two perpendicular lines OQ and OL in the XY-plane where OQ makes an angle with OX. Show that the principle moments at O are equal to

$$\frac{1}{2}\left\{A+B \pm \sqrt{(A-B)^2+4F^2}\right. .$$ 8

(b) Define principle axes. Find the conditions that Y-axis for given material system to be a principal axis at any point of its length ; also find the other two principal axes. What happens when an axis passes through the centre of gravity of the body ? Justify. 8

(c) State D'Alembert's principle and deduce the equation of motion of a rigid body. Show that the motion of a body about its centre of inertia is the same as it would be if the centre of inertia were fixed and the same forces acted on the body. 8

(d) Derive the expression for length of a simple equivalent pendulum of a compound pendulum. A solid homogeneous cone of weight h and vertical angle 2α, osicillates about a horizontal axis through its vertex. Show that the length of the simple equivalent pendulum is $\frac{h}{5}(4+\tan^2\alpha)$. 8

(e) Define degrees of freedom. Show that the rate of change of angular momentum of a rigid body about the axis of rotation is equal to the sum of moments about the same axis of all forces acting on the body. 8

2. Answer any *two* questions : 2×3

(a) Find the moment of inertia of a rigid circular cylinder about a straight line through its centre of gravity perpendicular to its axis.

(b) Show that the centre of suspension and centre of oscillation are convertible.

(c) Three equal uniform rods placed in a straight line are freely jointed at junctions and move with a velocity v perpendicular to their length. If the middle point of the middle rod be suddenly fixed, show that the ends of the other two rods will meet in time $\frac{4\pi a}{9v}$, where a is the length of the each rod.

Group—B

(Hydrostatics)

[Marks : 25]

3. Answer any *two* questions : 2×8

(a) Define a metacentre. Find the condition of existence of a metacentre of a body. Obtain the formula for finding the metacentre of a body following freely in a homogeneous liquid at rest under gravity. 8

(b) Prove that the pressure at any point in a fluid in equilibrium is the same in every direction. 8

(c) A semi-circular tube has its bounding diameter horizontal and contains equal volumes of n fiuid of densities successively equal to ρ, 2ρ, 3ρ,...arranged in this order. Show that if each fluid subtends an angle 2α at the centre, and the tube just holds them all, then $\tan n\alpha = (2n + 1) \tan \alpha$. 8

4. Answer any *three* questions : 3×3

(a) Explain the statement : "The surface for pond is horizontal, but the surface of an ocean or a sea is spherical". 3

(b) Prove that in a fluid at rest under gravity, horizontal planes are surfaces of equal density. 3

(c) Let a fluid be in equilibrium under the action of a given system of external forces. Then prove that the surfaces of equi-pressure are intersected orthogonally by the lines of force.

(d) Show that the equation for a gas in an adiabatic temperature change is $T\upsilon^{\gamma-1}$ = constant, the symbols having usual measuring. 3

(e) What happens to the position of the centre of pressure if the plane area is lowered infinitely ?

Group—C

(Discrete Mathematics)

[Marks : 20]

5. Answer any *one* question : 1×15

(a) (i) Define bounded lattice with an example. Prove that every chain is a distribution lattice. 2+3

(ii) Define generating function of a numerical function. Find the sequence represented by the closed form of the generating function 5

$$f(x)=\frac{1}{1-3x},\ \text{for } |3x|<1$$

(iii) Define Eulerian path and Eulerian circuit of a graph with an example for each. When is a graph called an Eulerian graph ?

(b) (i) Define Hasse diagram. Draw the Hasse diagram of the poset $\{S_{30}, D\}$, where S_{30} is the set of all divisors of 30 and D is the relation of 'division'. 4

(ii) Define tree and spanning tree. Show that every connected graph has atleast one spanning tree. 5

(iii) Prove that if a simple connected graph is planner then $e \leq 3n - 6$, where e is the number of edges and n is the number of vertices. 5

6. Answer any *one* question : 1×3

(a) Using of generating functions solve the recurrence relation $\partial_n = 2\partial_{n-1}$ for all $n \geq 1$ and $\partial_0 = 3$. 3

(b) Given that the value of $p \to q$ is true, can you determine the value of $\sim pV(p \leftrightarrow q)$. 3

7. Answer any *one* question : 1×2

(a) State the principle of inclusion and exclusion.

(b) Define poset and give an example.

Group—D

(Mathematical Modelling)

[Marks : 15]

8. Answer any *one* question : 1×15

(a) (i) The Lotka-Voltera competition model of two different species satisfy the differential equation

$$\frac{dx}{dt} = r_1 x - \alpha_1 xy$$
$$\frac{dy}{dt} = r_2 y - \alpha_2 xy$$

where are $r_1, r_2, \alpha_1, \alpha_2$ positive constants with $x(0) = x_0$ and $y(0) = y_0$ and $x(t)$, $y(t)$ are the population of different species at time t. Find its solution and interpret the system geometrically. 4+4

(ii) In a single species population, let $P(t)$ be the population size at time t, $b = b_1 - b_2 p$ be the birth rate and $d = d_1 + d_2 P$ be the death rate.

I. Find the population at time t.

II. Sketch the graph of $p(t)$ against t and discuss the behaviour of the solution with respect to the initial population. 3+4

(b) (i) Determine the nature of the critical point of the linear autonomous system

$$\dot{x} = x + y$$
$$\dot{y} = -x + y$$

Determine whether the critical point is stable or not. Draw a phase portrait of the system.

(ii) Write the basic-assumption of single species model when birth rate and death date are constants. Construct the differential equation of the model and solve it. Show that population grows or decays exponentially. 7+8
