Total No. of pages : 8 M/19/B.Sc/Part-II/Math.-V(H)

2019

Part-II

MATHEMATICS

(Honours)

Paper-V

[New Syllabus]

Full Marks - 90

Time : 4 Hours

The Questions are of equal value for any group/half.
The figures in the right-hand margin indicate marks.
Candidates are required to give their answers in their own words as far as practicable.
Illustrate the answers wherever necessary.

Group - A

(Real Analysis - II)

Marks : 50

1. Answer any **two** question : 15×2

(a) (i) If $f:[a,b]\to R$ is contimuous on $[a,b]$ then prove that f is Riemann integral on $[a,b]$ is the converse true ? justify.

(ii) State and prove fundamental theorem of

P.T.O.

integral calculus.

(iii) A function f is defined on [0.1] by

$$f(x) = \begin{cases} \sin x, \ x \text{ is rational} \\ x, \ x \text{ is irrational} \end{cases}$$

Evaluate $\overline{\int_{0}^{\frac{\pi}{2}}} f$, $\underline{\int_{0}^{\frac{\pi}{2}}} f$ and hence show that f is not R-integrable on $[0, \frac{\pi}{2}]$

(4+1)+(1+4)+(3+2)

(b) (i) Let the functions f: [a,b]→ R and g: [a,b]→R be both R-integrable on [a,b] then show that fg is R-integrable on [a,b].

(ii) If a functions f:[a,b]→R be integrable on [a,b] and the function F be defined by

$F(x) = \int_{a}^{x} f(t)dt,\ x \in [a,b]$ the prove that F(x) is differentiable at any point $c \in [a,b]$ at which f is continuous and $F'(c) = f(c)$

(iii) Show that $\iint_E \frac{dxdy}{(1+x^2+y^2)^2} = \frac{\sqrt{3}}{2} \tan^{-1} \frac{1}{2}$

5+5+5

(c) (i) Let $f(x) = \begin{cases} 1, \ 0 \le x \le 1 \\ x, \ 1 < x \le 2 \end{cases}$

Verify that the function F defined by

$F(x) = \int_{o}^{x} f(t)dt$, $x \in [0,2]$ is differentiable on $[0,2]$ and $F'(x) = f(x)$, $x \in [0,2]$

(ii) Let a be only point of singularity of f in [a,b] and f, g are both integrable on $[a+\in, b]$ for all $\in$ satisfying $o < \in < b-a$. If there exists a positive number λ such that $o < f(x) < \lambda g(x)\ \forall x \in (a,b]$ then prove that $\int_{a}^{b} f(x)dx$ converges if $\int_{a}^{b} g(x)dx$ converges and $\int_{a}^{b} g(x)dx$ diveeges if $\int_{a}^{b} f(x)dx$ diverges.

(3+2)+5+5

(iii) Evaluate $\int_{0}^{\frac{\pi}{2}} \log \sin x dx$.

2. Answer any **two** questins. 2×8

(a) (i) Let *(a,b)* be any point in the domain of defination of $f(x,y)$. Let $f_x(a,b)=0$ and $f_y(a,b)=0$ and, $f_{xx}(a,b)=A$, $f_{xy}(a,b)=B$, $f_{yy}(a,b)=C$. If AC - $B^2 > 0$ and A A < 0 then show that *f(a,b)* will be a maximum value of *f(x,y)*.

(ii) Discuss the convergens of

$$\int_0^{\frac{\pi}{2}} (\cos x)^l (\sin x)^l \, dx.$$ 4+4

(b) Prove that $\int_o^{\infty} e^{-mx} \frac{\sin nx}{x} dx = \tan^{-1}\frac{n}{m} (m > 0)$.

Hence evaluate $\int_o^{\infty} \frac{\sin nx}{x} dx$. 6+2

(c) (i) Show that the second mean value theorem (weierstrass form) is applicable to $\int_a^b \frac{\sin x}{x} dx$ where $o < a < b < \infty$. Also prove that $\left|\int_a^b \frac{\sin x}{x} dx\right| \leq \frac{4}{a}$.

(ii) Find the expansion of $f(x, y) = \sin(xy)$ in powers of $(x - 1)$ and $\left(y - \frac{\pi}{2}\right)$ up to and including second degree terms using Taylor's theoren. 4+4

3. Answer any **one** question. 1x4

(a) If 0 < p < 1 show that $\Gamma(p)\Gamma(1-p) = \int_o^{\infty} \frac{x^{p-1}}{1+x} dx$. 4

(b) Show that $\lim_{x\to 0}\frac{\int_0^{x^2}\sin\sqrt{t}\,dt}{x^3}=\frac{2}{3}$. 4

Group - B

(Metric space)

Marks - 15

4. Answer any one question : 1×8

(a) (i) Show that the set l_∞ of all bounded sequencs $\{x_n\}$ fo real numbers with function 'd' defined by $d(\{x_n\},\{y_n\}) = \sup\left\{\left|x_n - y_n\right| : n \in N\right\}$ for all $\{x_n\}\{y_n\} \in l\infty$ is a metric space.

(ii) Prove that the discrete space (x, d) is a complete metric space. 4+4

(b) (i) Let (X,d) be a metric space, Prove that the union of an arbitrary collection of open sets of X is open.

(ii) Let A and B be two sets in a metric space (X, d). Show that $A \subseteq B \Rightarrow \bar{A} \subseteq \bar{B}$ when $\bar{A}$ and $\bar{B}$ are the closure of the set A and B.

4+4

5. Answer any **one** question : 1×4

(a) For any two points x, y, a, b in a metric space (X,d), show that

$$|d(x,y)-d(a,b)| \le d(x,a)+d(y,b).$$

(b) Define cauchy sequence in a metric (space, Prove that every convergent sequence is a cauchy sequence.

6. Answer any **one** question. 1×3

(a) Define a decreasing sequence of non empty sabsets of a metric space (X, d) State the Cantor's intersection theorem. (1+2)

(b) Prove that a finite set has no limit point 3

Group - C

(Complex Analysis)

Marks - 10

7. Answer any **one** question : 1×8

(a) (i) Show that harmonic function u(x, y) satisfies the differential equation $\frac{\partial^2 u}{\partial z \partial \overline{z}} = 0$ when $z = x + iy$.

(ii) Using Milne's method find the analytic function whose real part is

$$e^{-x}\left\{\left(x^2-y^2\right)\cos y+2xy\sin y\right\}$$ 4+4

(b) (i) If a function $f(x,y) = u + iv$ is differentiasle at a point $z_o = x_o + iy_0$. Then show that the partial derivatives u_x, u_y, v_x, v_y exists and $u_x = v_y$, $u_y = -v_x$ at the point (x_o, y_o).

(ii) Show that $f(z)=\overline{z}$ is continuous at $z=z_o$ but is not analytic at $z = z_o$. 5+3

8. Answer any **one** questions. 1×2

(a) Show that the real and imaginary parts of a analytic function are harmonic functions. 2

(b) If f is analytic function prove that 2

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left|f(z)\right|^2=4\left|f'(z)\right|.$$

Group - D

(Tensor Calculus)

Marks-15

9. Answer any **one** question. 8×1

(a) (i) If B_{ij} are components of a covarient tensor of second order and C^i, D^j are components of two contravanient vectors, show that Bij C^i D^j is an invariant.

(ii) Prove that $\left\{ \begin{matrix} i \\ i \; j \end{matrix} \right\} = \frac{\partial}{\partial x^j}\left(\log \sqrt{g}\right)$ where $g = |g_{ij}|$

4+4

(b) (i) If $B_{ij} = A_{ji}$, where A_{ij} is a covariant tensor, show that B_{ij} is a tensor of order 2

(ii) If a^{ij} is a Symmetic tensor, show that

$$a^{jk}[ij,k] = \frac{1}{2} a^{jk} \frac{\partial g_{jk}}{\partial x^i}$$ 4+4

10. Answer any **one** question 4×1

(a) Prove that the fundamental tensors behave in covariant differentiation as though they are constant. 4

(b) For the cuvatime tensor R^l_{ijk}, prove that

$R^l_{ijk} + R^l_{jki} + R^l_{kij} = 0$. 4

11. Answer any one question : 3×1

(a) What is quotient law. Define inner product of tensors. 1+2

(b) If $x^i = a_p{}^i, y^p$ and $z^i = b^i_q x^q$ show that $z^i = b^i_p a^p_q y^2$

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