2019

Part-II

MATHEMATICS

(Honours)

Paper-IV

[New Syllabus]

Full Marks - 90

Time : 4 Hours

The Questions are of equal value for any group/ half. The figures in the right-hand margin indicate marks. Candidates are required to give their answers in their own words as far as practicable. Illustrate the answers wherever necessary.

Group – A

(Analytical Dynomics of Particles)

Marks : 40

1. Answer any **one** question : 1×15

(a) (i) A particle of mas m on a straight line is attrached towards the origin on it with a force $m\mu$ *times the* distance from it and the resistance to motion at any point is mk times the square of the velocity there. If it starts from rest at a distance 'a' from the origin, prove that it will come to rest again at a distance 'b' from the origin, where

$(1+2ak)e^{-2ak} = (1-2bk)e^{2bk}$ 8

(ii) Over a small smooth pulley is placed a uniform flexible cord, the later is initially at rest and lengths $\ell - a$ and $\ell + a$ hang down on two sides. The pulley is now made to move with contant upward accederation f. show that the string will leave the pully after time

$\sqrt{\frac{\ell}{f+g}} \cos h^{-1} \frac{\ell}{a}$ 7

(b) (i) A particle mover with a central acceleration $\mu\left(r+\frac{a^4}{r^3}\right)$ being projected from an apse at a distance a with a velocity $2a\sqrt{\mu}$, prove that its path is $r^2\left(2+\cos\sqrt{3}\theta\right)=3a^2$ 8

(ii) If a planet was suddenly stopped in its orbit supposed circular, show that it would fall into the sun in a time which is $\sqrt{2}/8$ times the period of the planet's resolution. 7

2. Answer any **two** questions. 2×8

(a) A particle is acted on by a force parallel to the axis of y where acceleration is ky and is initially projected with a velocity $a\sqrt{k}$ parallel to the axis of x at a point where y = a; prove that it will describe a catenary.

(b) If the resistance varies as the velocity and the range on the horizontal plane through the point of projection is a maximum, show that the angle α which the direction of projection makes with the vertical is given by

$$\frac{\lambda(1+\lambda\cos\alpha)}{\cos\alpha+\lambda}=\log(1+\lambda\cos\alpha)$$ where λ is the ratio of the velocity of projection to the terrminal velocity.

(c) A spherical raindrop, falling freely, receives in each instant an increase of volume equal to λ times its surface area at that instant, show that the velocity at the end of time t and the distance fallen through in that time are respectively

$$\frac{g}{4\lambda}\left[(a+\lambda t)-\frac{a^4}{(a+\lambda t)^3}\right] \text{ and } \frac{gt^2}{8}\left[\frac{2a+\lambda t}{a+\lambda t}\right]^2$$

Where 'a' is the *initial radius of th raindrop* .

3. Answer any **three** questions : 3×3

(a) If v be the speed of a particle at any time t moving along x-axis and $v^2 = 4 - x^2$, then prove that the motion is S. H. M. Find the period of oscillation. 2+1

(b) A particle describes a parabola $y^2 = 4ax$ under a force which is directed perpendicular towards its axis, find the law of force. 3

(c) State Kepler's laws of planetary motion. 3

Group – B

(Analytical Statics)

Marks – 30

4. Answer any **three** questions. 3 × 8

(a) Three forces P, Q, R act along the sides of a triangle formed by the lines $x + y = 3$, $2x + y = 1$ and $x - y + 1 = 0$. Find the equation of the line of action of the resultent

(b) (i) Find the C. G. of the area enclosed by the curved $x^2 + y^2 = 2ax$ and $x^2 + y^2 = 2bx$ $(b<a)$ on the 1st quadrant. 4

(ii) Four equal rods each of weight W form a rhombus ABCD with smooth hinges at the joints. The frame is suspended by the end A and a weight W' is attached at C. A stiffening rod of negligible weights joins the middle points of AB, AD keeping these inclined at an angle α to AC. Show that the thrust on the stiffening rod is $(4W + 2W')\tan\alpha$. 4

(c) The ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ is placed with the axis of x vertical and its surface is rough. Show that a heavy particle will rest on it everywhere above its intersection with the cylinder $\frac{y^2}{b^2}\left(1+\frac{a^2}{b^2\mu^2}\right)+\frac{z^2}{c^2}\left(1+\frac{a^2}{\mu^2c^2}\right)=1$, where μ is the co-efficient of friction. 8

(d) State Principle of Virtual work for a particle. The middle points of opposite sides of a Quadrilate ral formed by four freely jointed weightless bars are connected by two light rods of length 'a' and 'b' in a state of tension. If T_1 and T_2 be the tensions of those rods, prove that 1+7

$$\frac{T_1}{a} > \frac{T_2}{b} = 0$$

e) What are ment by stable unstable eqilibrium of a body. A Body rests in equilibrium on another fixed body there being enough fricition to prevent sliding. The position of the two bodies in contact are spherical of radii 'r' and 'R' and the line of joining their centres in the position of equilibrium is vertical. Show that the equilibrium is stable Provided

$$\frac{1}{h} > \frac{1}{r} + \frac{1}{R}$$

Where h is the height of C.G. of the upper body in position of equilibrium above the point of contact.

2+6

5. Answer any **two** questions : 3×2

(a) Discuss the significance of virial 3

(b) State laws of friction. 3

(c) Prove that for any given system of forces can have only one central anis. 3

Group – C

(Differential Equation-II)

Marks - 20

6. Answer any **one** question : 1×15

(a) (i) Find the power series solution of the initial value problem

$$4\frac{d^2y}{dx^2}-4\frac{dy}{dx}+y=0 \quad \text{about } x = 2 \text{ with}$$

$$y(2) = 0,\; y'(2)=\frac{1}{e}$$ 8

(ii) State convolution theorem for Laplace Transformation. 2

(iii) Solve the following :

$z(x + y)p + z(x - y)\,q = x^2 + y^2$ 5

where $p=\frac{\partial z}{\partial x},\; q=\frac{\partial z}{\partial y}$

(b) (i) Solve : $\frac{dx}{dt}+4x+3y=t$

$\frac{dy}{dt}+2x+5y=e^t$ 6

(ii) Using convolution theorem, evaluate

$$L^{-1}\left\{\frac{1}{p(p+1)^3}\right\}$$ 4

(iii) Use Laplace transform to solve the following differential equation

$$y'' + 2y' + 2y = 2, \; y(0) = 0, \; y'(0) = 1 \qquad 5$$

7. Answer any **one** question : 3×1

(a) Locate and classify the singular points of the equation $x^3(x-1)y'' + 2x^4y' + 4y = 0$. 3

(b) Form a Partial Differential Equation by eleminating the function f from $z = xf\left(\frac{y}{x}\right)$ 3

8. Answer any **one** question : 2×1

(a) Evaluate $L^{-1}\left\{\int_{o}^{\infty} \frac{1}{p(p+1)} dp\right\}$

(b) What is semi-linear partial differential equation? Give an example. 2
