2019

Part – II

MATHEMATICS

(Honours)

Paper-III

[New Syllabus]

Full Marks - 90

Time : 4 Hours

The Questions are of equal value for any group/half.
The figures in the right-hand margin indicate marks.
Candidates are required to give their answers in their own words as far as practicable.
Illustrate the answers wherever necessary.

Group – A

(Vector Analysis)

Marks : 25

1. Answer any **one** question : 8×1

(a) (i) Let $\vec{a},\vec{b},\vec{c}$ and $\vec{d}$ be four vectors. Prove that

$$(\vec{a}\times\vec{b}).(\vec{c}\times\vec{d})+(\vec{b}\times\vec{c}).(\vec{a}\times\vec{d})+(\vec{c}\times\vec{a}).(\vec{b}\times\vec{d})=0$$

Hence deduce that

$$\sin(\alpha-\beta)\sin(\alpha+\beta)=\sin^2\alpha-\sin^2\beta$$

(ii) Find the projection of $\vec{a} = 2\hat{i} + \hat{j} - 3\hat{k}$ on $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$. (3+3+2)

(b) (i) State and prove Green's theorem.

(ii) Find the directional derivative of A^2, where $\vec{A} = xy^2\hat{i} + zy^2\hat{j} + xz^2\hat{k}$ at the point (2,0,3) in the direction of the outward normal to the curve $x^2 + y^2 + z^2 = 14$ at the point (3, 2, 1). 4+4

2. Answer any **three** questions : 4×3

(a) Verify divergence theorem for $\vec{A} = 4x\hat{i} - 2y^2\hat{i} + z^2\hat{k}$ taken over the region bounded by $x^2 + y^2 = 4$, $z = 3$ and $z = 6$.

(b) Show that the Frenet Serret formula can be expressed as $\frac{d\hat{t}}{ds} = \vec{w} \times \hat{t}$, $\frac{d\hat{n}}{ds} = \vec{w} \times \hat{n}$, $\frac{d\hat{b}}{ds} = \vec{w} \times \hat{b}$ where $\vec{w}$ is to be found by you.

(c) Evaluate $\iint_S \vec{A}.\hat{n}ds$, where $\vec{A} = 2x\hat{j} + y\hat{i} - z\hat{k}$ and S is the surface of the plane $2x + y = 6$ included in the first octant cut off by the plane $z = 4$.

(d) A force of 15 units acts is the direction of the vector $\hat{i}-2\hat{j}+2\hat{k}$ and passes through a point $2\hat{i}-2\hat{j}+2\hat{k}$, Find the moment of the force $\hat{i}+2\hat{k}$ anout the point $\hat{i}+\hat{j}+\hat{k}$.

(e) If $\vec{A}$ be a vector point function defined in the region enclosed by a surface S such that its finite order partial derivatives be finite, single valued and continuous throughout V and S, then

$\iiint_V (\vec{\nabla}\times\vec{A})\,dv = \iint_S (\hat{n}\times\vec{A})\,ds$ $\hat{n}$ has usual meanings.

3. Answer any **one** question : 3×1

(a) For the space curve $x = t$, $y = t^2$, $z=\frac{2}{3}t^3$, determine the curvature at the point where $t = 1$.

(b) Show that if $\phi(x,y,z)$ is any solution of Laplace's equation, then $\vec{\nabla}\phi$ is a vector which is both solenoidal and irrotational.

4. Answer any **one** question : 2×1

(a) State the Divergence Theorem of Gauss in vector Integration.

(b) Find the equation of the plane containing the line $\vec{r}=t\vec{\alpha}$ and is perpendicular to the plane containing the lines $\vec{r}=t_1\vec{\beta}$ and $\vec{r}=t_2\vec{\gamma}$.

Group - B
(Analytical Geometry-3D)
30 Marks

5. Answer any **one** question : 15×1

(a) (i) Show that the equation to the plane containing the straight line

$$\frac{y}{b}+\frac{z}{c}=1,\ x=0$$

and parallel to the straight line

$\frac{x}{a}-\frac{z}{c}=1$, y = 0 is $\frac{x}{a}-\frac{y}{b}-\frac{z}{c}+1=0$ and if 2d be the shortest distance between the lines, then show that

$$\frac{1}{d^2}=\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}$$

(ii) Reduce the equation $x^2+y^2+z^2-2yz+2zx-2xy+x-4y+z+1=0$ to the cononical form and state the nature of the surface.

(b) (i) Show that the generators of the hyperboloid $\frac{x^2}{25}+\frac{y^2}{16}-\frac{z^2}{4}=1$ which are parallel to the plane $4x-5y-10z+7=0$ are

$$x+5=0,\ y+2z=0 \text{ and } y+4=0,\ 2x=5z$$

(ii) Find the locus of the vertices of the right circular cones that pass through the elipse

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, z = 0,

6. Answer any **one** question : 8×1

(a) The plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ meets the coordinate axes in A, B, C. Find the equation of the cone generated by lines drawn from origin to meet the circle ABC.

(b) Find the equation of the sphere having the circle $x^2 + y^2 + z^2 = 9$, $x - 2y + 2z = 5$ for a great circle, Also deternine its centre and radius.

7. Answer any **one** question : 4×1

(a) Let the perpendiculars SL, SM, SN be drawn from the point S (a, b, c) to the co-ordinate planes. Find the equation of the plane LMN.

(b) Find the equation to the right circular cylinder whose radius is 5 and whose axis is

$$\frac{x-y}{2}=\frac{y-2}{3}=\frac{z-3}{4}$$

8. Answer any **one** question : 3×1

(a) Find the shortest distance between the lines $x = y = z$ and $x + y = z$, $z - x = 2$

(b) Find the position of the center of the conicoid

$14x^2 + 14y^2 + 8z^2 - 4yz - 4zx - 8xy + 18x - 18y + 5 = 0$

Group - C

(LPP and Game theory)

35 Marks

9. Answer any **one** question. 15×1

(a) (i) Prove that every extreme point of the convex set of all feasible solutions of the system Ax = b, $x \geq 0$ corresponds to a basic feasible solutions. 8

(ii) Solve the following LPP : 7

Maximize $z = 2x_1 - 3x_2$

s. t.

$$x_1 \leq 4$$

$$x_2 \leq 6$$

$$x_1 + x_2 \leq 5$$

$$-x_2 \leq -1$$

$$x_1, x_2 \geq 0$$

(b) (i) $x_1 = 2, x_2 = 3, x_3 = 1$ is a feasible solution of the following LPP : 7

Maximize $z = x_1 + 2x_2 + 4x_3$

s. t.

$$2x_1 + x_2 + 4x_3 = 11$$

$$3x_1 + x_2 + 5x_3 = 14$$

$$x_1, x_2, x_3 \geq 0$$

(ii) Find BFS :

If x be any feasible solution to the primal problem Max $z = CX$, $AX \le b$, $X \ge 0$ and V be any feasible solution to the dual problem then prove that $CX \le b^T V$ 8

10. Answer any **two** question 8 × 2

(a) Using two phase method, solve the LPP :

Min. $z = 3x_1 + 2x_2$

s.t. $2x_1 + x_2 \ge 14$

$2x_1 + 3x_2 \ge 22$

$x_1 + x_2 \ge 1$

$x_1, x_2 \ge 0$

(b) Find the minimum cost of transportation problem :

	D_1	D_2	D_3	a_1
O_1	4	3	2	10
O_2	1	5	0	13
O_3	3	8	6	12
bj	8	5	4	

Symbols have their useual meanings.

(c) Solve the game problem by simplex method :

$$\begin{array}{c c} & B \\ A & \begin{bmatrix} 1 & -1 & 3 \\ 3 & 5 & -3 \\ 6 & 2 & -2 \end{bmatrix} \end{array}$$

11. Answer any **one** question : 4×1

(a) Solve graphically the following game problem

		B			
		B_1	B_2	B_3	B_4
A	A_1	6	5	2	3
	A_2	1	2	6	3

(b) Prove that if a constant be added to any row or any column of the cost matrix of an assigment problem then the resulting assignment problem has the same optimal solution as the original problem.
