

2012**MCA****1st Semester Examination****FOUNDATION IN MATHEMATICS AND LOGIC****PAPER—MCA-104***Full Marks : 100**Time : 3 Hours**The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.**Illustrate the answers wherever necessary.***Answer Q. No. 1 and any five from the rest**

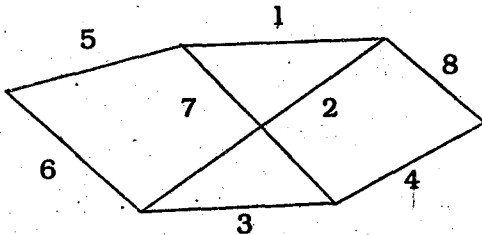
1. Answer any five questions : 5×2
- (a) Define power set. Explain it with an example. 1+1
- (b) Define mapping. Explain it with an example. 1+1
- (c) Define duality principle of Boolean algebra. 2
- (d) Show that any matrix can be expressed as a sum of symmetric and skew-symmetric matrix. 2
- (e) Define Hamiltonian graph. Explain it with an example. 1+1
- (f) Define partial sum sequence of a series. What is relation between series and partial sum sequence? 1+1

(Turn Over)

2. (a) Define Cartesian product of two sets. Show that

$$A \times (B \cap C) = (A \times B) \cap (A \times C). \quad 1+5$$
- (b) State and prove the Euler's theorem related to planar graph. 1+5
3. (a) State induction principle. Use mathematical induction to show that

$$1 + 2 + 2^2 + \dots + 2^n = 2^{n+1} - 1$$
for all non-negative integers.
- (b) Demonstrate Dijkstra's algorithm for the graph : 6



4. (a) Define inverse mapping. Show that the mapping $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 3x + 1, x \in \mathbb{R}$ is invertible and find its inverse. 1+5
- (b) Define Rank and Nullity of a graph, and binary tree. Show that number of internal vertices in a binary tree is one less than the number of pendant vertices. 3+3

5. (a) Define bounded sequence. Show that the sequence $\{x_n\}$ is convergent, where $x_1 = \sqrt{6}$ and

$$x_n = \sqrt{6 + x_{n-1}}, \quad \forall n \geq 1. \quad 1+5$$

- (b) Define Eulerian graph. Prove that a graph G is an Eulerian graph if and only if all vertices of G are even degree. 1+5

6. (a) Define adjoint of a square matrix. Find the inverse

of the matrix $\begin{pmatrix} 2 & 4 & 0 \\ 4 & 9 & 5 \\ 1 & 3 & 7 \end{pmatrix}$. 1+5

- (b) The English alphabet contains 21 consonants, and 5 vowels. How many strings of six lower case letters of English alphabet contain :

- (i) exactly one vowel ;
 (ii) exactly two vowels ;
 (iii) at least one vowel. 2+2+2

7. (a) Prove without expanding :

$$\begin{vmatrix} 1 & a & a^2 & a^3 + bcd \\ 1 & b & b^2 & b^3 + cda \\ 1 & c & c^2 & c^3 + dab \\ 1 & d & d^2 & d^3 + abc \end{vmatrix} = 0. \quad 6$$

(b) In a Boolean algebra B,

Show that (a) $(a + b)' = a' \cdot b'$

(b) $(a \cdot b)' = a' + b'$

for all $a, b \in B$.

6

8. (a) Define and draw the graphs of the following function floor function, ceiling function and logarithmic function. 2+2+2

(b) Define and draw Venn-diagram of the following set operations :

Union, Intersection, Complement, Symmetric difference.

$$1\frac{1}{2} + 1\frac{1}{2} + 1\frac{1}{2} + 1\frac{1}{2}$$

[Internal Assessment — 30]
