2011

MCA

1st Semester Examination

FOUNDATION IN MATHEMATICS AND LOGIC

PAPER—MCA-104

Full Marks : 100

Time : 3 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Answer Q. No. *1* and any *five* from the rest

1. (a) State the duality principle of Boolean Algebra. 1×10

(b) Define complete graph.

(c) Draw a graph whose degree sequence of vertices is {1, 2, 2, 4, 5}.

(d) Give the example of two non-null matrices, such that their product is a null matrix.

(e) Write down the truth table in connection with 'OR' logic.

(f) Prove or disprove $f(x) = x^2 - 1$ is one-one.

(g) Define proposition on logic.

(h) Give an example of a relation, which is reflexive, symmetric but not transitive.

(i) Draw the logic gate for

$$F = XY + \overline{X}Y + X\overline{Y}.$$

(j) Define Bipartite graph.

2. (a) For the sets A, B, C, show that

$$A \times (B \cup C) = A \times B) \cup (A \times C)$$

(b) Define tautology,

Show that, the truth values of $p \leftrightarrow q$ and

$(p \rightarrow q) \wedge (q \rightarrow p)$ are equivalent.

(c) By mathematical induction, prove that,

$$1^2 - 2^2 + 3^2 - 4^2 + \ldots\ldots + (-1)^{n-1} n^2 = (-1)^{n-1} \frac{n(n+1)}{2}$$

4+4+4

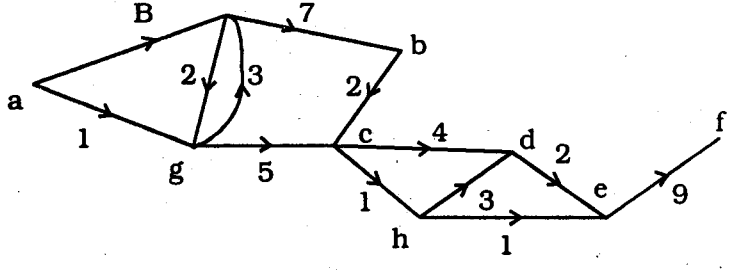
3. (a) A simple graph with n-vertices an k components can have at most $(n-k)(n-k+1)/2$ edges—prove it.

6

(b) Define tree and binary tree. Find the no of pendent vertices of a binary tree with n-vertices.

(2+4)

4. (a) Describe the Dijkstra algrorithm to find the shortest pat. Hence find the spanning tree of the following graph : 4+4



(b) Expand the matrix $A = \begin{pmatrix} 2 & 4 & 0 \\ 1 & 3 & 9 \\ 4 & 6 & 1 \end{pmatrix}$ as a sum of symmetric & skew-symmetric matrices. 4

5. (a) Define idempotent matrix. If AB = A and BA = B, for the matrices A, B, then show that A & A^T, B & B^T are idempotent.

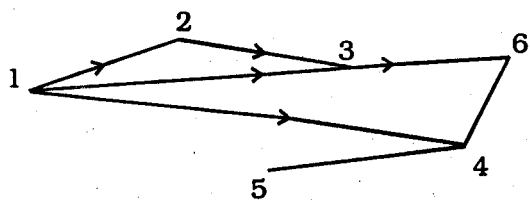
(b) Simplify the Boolean function $f(x, y, z) = \sum(1, 5, 7)$ and draw the logic gate. 6+6

6. (a) If $\left|\vec{a}+\vec{b}\right| = \left|\vec{a}-\vec{b}\right|$, then show that $\vec{a}$ & $\vec{b}$ are perpendicular. 6

(b) Define NAND-logic gate. Show that it is an universal gate. 6

7. (a) Determine the nature of relation ρ defined on the set of integers by $a\rho b$ iff $a - b = 0$. 6

(b) Formulate the incidence matrix for the following simple graph —



(c) Explain XOR of two input bits a and b is equivalent to a + b mod 2. 4+4+4

8. (a) By Karnaugh map, simplify

$X = \overline{AB} + \overline{A}B + A\overline{B}$ 6

(b) Show that, the sequence $\{x_n\}$ converges to 3, where $x_{n+1} = \sqrt{6 + x_n}$, $n > 1$, $x_1 = \sqrt{6}$. 6

[Internal Assessment — 30]

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