2019

MSc

4^{th} Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER – MTM-401

Full Marks : 50

Time : 2 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

(Notations and symbols have their usual meanings)

Answer Q. No. ***1*** and any ***FOUR*** from Q. No. ***2*** to Q. No. ***6***

1. Answer any ***FOUR*** questions from the following. 2x4

a)If E_1 is an open set in normed linear space X and $E_2 \subset X$,then show that $E_1 + E_2$ is open in X .

b) Let X be a normed linear space and M be a subspace of X . Prove that for any real or complex number K , for any $x \in X$ and $y \in M$ we have $\| kx+y \| \geq |k|$ dist (x,M)

c) Let X be a normed linear space. $x_n \to x$ weakly in X implies $x_n \to x$ strongly in X. Examine it in general .

d) Show that every normed linear space can be embedded as a dense subspace of a Banach space.

e) If in an inner product space X, $\langle x, u\rangle = \langle x, v\rangle$ for all $x \in X$, show that u = v.

f) Let H be a Hilbert space and A, B ∈ BL (H) be two normal operators. If A commutes with B* and B commutes with A* then prove that A+B and AB are normal .

2. a) Prove that a normed linear space X is a Banach space if every absolutely summable series of elements in X is summable in X .When is a normed linear space said to be uniformly convex.

b) Prove that every inner product space X is uniformly convex in the norm $\| \; \|$. 4+(1+3)

3. a) State Pythagorean theorem in an inner product space

b) Let { u_1, u_2, ...} be a countable ortho normal set in an inner product space X and $x \in X$. Prove that $\sum_n |\langle x, u_n\rangle|^2 \leq \|x\|^2$

Also show that equality holds if and only If $x = \sum_n \langle x, u_n\rangle u_n$ 2+(4+2)

4. a) Let $X = C[0,1]$ with the supremum norm. Consider the sequence $\{x_n\}$ in X where

$x_n(t) = \frac{t^n}{n^3}$, $t \in [0,1]$. Check whether the series $\sum_{n=1}^{\infty} x_n$ is summable in X.

b) Let X and Y be Banach spaces and $A \in BL(X,Y)$. Show that there is a constant $c > 0$ such that $\|Ax\| \geq c\|x\|$ for all $x \in X$ if and only if Ker(A) = {0} and R(A), the range of A is closed in X. (4+4=8)

5. a) State and prove the Riesz representation theorem.

b) Let $S \in BL(H)$, where H is a Hilbert space. Prove that for all $x,y \in H$,

$< Sx,y >= \frac{1}{4}\sum_{n=0}^{3} i^n < S(x+i^n y), (x+i^n y) >$. 5+3

6. a) Let H be a Hilbert space, $T \in BL(H)$ and $< Ax, x >=0$ for all $x \in H$. Then show that $A = 0$. Also, show by an example that this result is not true for a real Hilbert space.

b) Let H be Hilbert space and $E \subset H$ Prove that $\overline{\text{span}}(E^{\perp\perp}) = E^{\perp\perp}$. 5+3

[Internal Assessment: 10 Marks]