2019

MSc

2nd Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER – MTM-205

Full Marks : 50

Time : 2 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

(General Theory of Continuum Mechanics)

Answer question No. *1* and any ***FOUR*** from the rest

(1). Answer any ***FOUR*** questions : 4x2

(i) Find the relation between α and β such that the small deformation defined by $u_1 = \alpha x_1 + 3x_2, u_2 = x_1 - \beta x_2$ and $u_3 = 3x_3$ is isochoric.

(ii) Prove that the extension of a line element through the centre of strain quadric in the direction of any central radius vector is equal to the inverse of the square of the radius vector.

(iii) The components of a stress dyadic at a point referred to the (x_1, x_2, x_3) system are given by

$$(T_{ij}) = \begin{pmatrix} 12 & 9 & 0 \\ 9 & -12 & 0 \\ 0 & 0 & 6 \end{pmatrix}$$

For this state of stress, determine the maximum shear stress.

(iv) Define principal stress and principle direction of stress.

(v) Show that the difference between two values of a stream function at two points represents the flux of a fluid across any curve joining these two points.

(vi) Differentiate between stream line and path line.

(vii) Define green elastic material.

(viii) When is a material called isotropic and anisotropic?

(2) Answer any ***FOUR*** questions: 4x4

(i) The displacement field for small deformation is given by 4

$$u_1 = (X_1 - X_3)^2 \qquad u_2 = (X_1 + X_3)^2 \qquad u_3 = -X_1 X_2$$

Determine rotation tensor and rotation vector at the point (0.2,-1).

(ii) Discuss the volumetric strain for small deformation of a body. 4

(iii) For the following stress distribution. 4

$$(T_{ij}) = \begin{pmatrix} x_1 + x_2 & T_{12} & 0 \\ T_{12} & x_1 - 2x_2 & 0 \\ 0 & 0 & x_2 \end{pmatrix}$$

Find $T_{12}(x_1, x_2)$ in order that stress distribution is in equilibrium with zero body force and the stress vector on $x_1 = 1$ is given by

$\vec{T}^n = (1 + x_2)\,\vec{e}_1 + (2 - x_2)\,\vec{e}_2$.

(iv) Establish the stress vector and stress tensor relationship. 4

(v) Show that the following is the possible form of the boundary surface of a liquid motion. 4

$$\frac{x^2}{a^2}\tan^2 t + \frac{y^2}{b^2}\cot^2 t = 1$$

(vi) 'The rotation in a fluid can be measured by $\nabla \times \vec{V}$ where $\vec{V}$ be the velocity of the fluid.' – Explain it. 4

(vii) Show that the principal directions of strain at each point in a linearly elastic isotropic body must be coincident with the principle directions of stress. 4

(viii) Deduce the stress – strain relations for a linear elastic solid. 4

(3) Answer any ***TWO*** questions: 2x8

(i) Derive the maximum shearing strain at any point of the continuum in terms of principal strains at that point. 8

(ii) (a) Find the integral of the Euler equation of motion for perfect fluid stating necessary assumptions when the flow is rotational and steady. 6

(b) If $\phi = (x_1 - t)(x_2 - t)$ be the velocity potential of a two-dimensional irrotational motion of a continuum , then show that stream lines at time t are $(x_1 \quad t)^2 \quad (x_2 \quad t)^2$ = constant 2

(iii) In case of motion of liquid in a part of a plane bounded by a straight line due to a source in the plane prove that if mρ is the mass of the liquid of density ρ generated at the source per unit of time, the pressure on the length 2l of the boundary immediately opposite to the source is less than that on an equal length at a great distance by

$$\frac{m^2\rho}{2\pi^2}\left(\frac{1}{c}\tan^{-1}\frac{l}{c} - \frac{l}{l^2+c^2}\right)$$

where c is the distance of the source from the boundary. 8

(iv) Derive the basic elastic constants for isotropic elastic solid. 8

[Internal Assessment – 10 marks]