**2019**

**MSc**

**2nd Semester Examination**

# APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

**PAPER – MTM-203**

**Full Marks : 50**

**Time : 2 Hours**

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

## (Unit-I)

# (Abstract Algebra)

1. Answer any *TWO* questions: 2x2

   a) Define group action with an example.

   b) Write down the class equation of a finite group.
   Is 1+1+3+5=10, a class equation for a group of order 10? Justify your answer.

   c) Consider the group G = $Z_4 \times Z_6$. Let H = $\langle(\bar{0},\bar{2})\rangle$. Compute $\frac{G}{H}$.

   d) Show that a group of order 87 is abelian.

2. Answer any *TWO* questions: 2x4

   a) Define solvable group. Show that subgroup of a solvable group is solvable.

   b) Find the maximal ideals and the prime ideals in the ring $(\mathbb{Z}_{12}, +, .)$.

   c) State Sylow's third theorem. Show that no group of order 63 is simple.

   d) If G is an abelian group having subgroup $H_1, H_2, ..., H_t$ such that $|H_i \cap H_j| = 1$, for all $i \neq j$, then show that $K = H_1 H_2 ... H_t$ is a subgroup of G of order $|H_1| \times |H_2| \times ... \times |H_t|$ and $K \cong H_1 \times H_2 \times ... \times H_t$.

3. Answer any *ONE* question: 1x8

   a) i) Show that in the ring $(\mathbb{Z} \times \mathbb{Z}, +, .)$, the set $I = \{(x, 0) : x \in \mathbb{Z}\}$ is a prime ideal but not a maximal ideal. (5)

   ii) Let $K \subseteq F$ be a field extension and $\alpha \in F$ be algebraic over K. Then show that there exists a unique monic irreducible polynomial $f(x) \in K[x]$ such that $f(\alpha) = 0$. (3)

   b) i) Suppose G be a group of order $p^2 q$ where p and q are distinct primes. Then show that G is not simple. (4)

   ii) If $K \subseteq F \subseteq L$ is a tower of fields, then show that (4)

   $$[L : F][F : K] = [L : K]$$

   Where [ L : F ] denotes the degree of L over F.

**[ Internal Assessment : 5 Marks ]**

# (Unit-II)

# (Linear Algebra)

**(Notation and symbols have their usual meaning)**

1. Answer any ***TWO*** questions: 2x2=4

   a) Define a Quotient space in linear algebra.
   b) Define a linear functional on a vector space with an example.
   c) Let T be a linear operator on finite-dimensional vector space V. When T is said to be diagonalizable?
   d) Define Jordan Block with an example or state Spectral Theorem in linear algebra.

2. Answer any ***TWO*** questions: 4x2=8

   a) Find all possible Jordan canonical forms for a linear operator T: V → V, where V is a finite dimensional vector space whose characteristic polynomial is $(t-2)^3\ (t-5)^5$. In each case, find the minimal polynomial m(t).
   b) State and prove First Isomorphism Theorem in linear algebra.
   c) Let T: $M_{2\times3}(F) \to M_{2\times2}(F)$ be defined by $T\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} = \begin{pmatrix} 2a_{11} - a_{12} & a_{13} + 2a_{12} \\ 0 & 0 \end{pmatrix}$.

      Find the Null space of T and Range space of T. Determine whether T is one-to-one or onto.

   d) Let P(t) be a minimal polynomial of a linear operator T on a finite dimensional vector space V. Then Prove the following.
      i) for any polynomial g(t), if g(T)= $T_0$, then P(t) divides g(t). In particular, p(t) divides the characteristic polynomial of T.
      ii) the minimum polynomial of T is unique.

3. Answer any *ONE* question. 1x8=8

(a) (I) Let T be a linear operator on a finite dimensional vector space V and let c be a scalar.

Then prove that the following are equivalent:

(i) c is a characteristic value of $T$.

(ii) The operator $T-cI$ is singular (not invertible).

(iii) Det $(T-cI)=0$.

(II). Let $V$ be the vector space of all polynomial functions $p$ form $\mathbb{R}$ into $\mathbb{R}$ ( $\mathbb{R}$ be the set of real numbers) which have degree 2 or less. Define three linear functional on $V$ by $f_1(p) = \int_0^1 p(x)dx, f_2(p) = \int_0^2 p(x)dx, f_3(p) = \int_0^{-1} p(x)dx.$

Show that $\{f_1, f_2, f_3\}$ is a basis of $V^*$( dual basis). Determine a basis for $V$ such that $\{f_1, f_2, f_3\}$ is its dual basis. (3+5=8)

b) (i) Let $V$ and $W$ be finite-dimensional vector spaces with ordered bases $\beta$ and $\gamma$, respectively. Let $T:V\to W$ be linear. Then show that $T$ is invertible if and only if $[T]_\beta^\gamma$ is invertible. Further show that $[T^{-1}]_\gamma^\beta = ([T]_\beta^\gamma)^{-1}$

(ii) Define the characteristic value and characteristic vector of a linear operator on a vector space. (6+2=8)

[ Internal Assessment: 05 Marks.]