PG/IIIS/MTM-306/19

M.Sc. 3rd Semester Examination, 2019

MATHEMATICS

PAPER—MTM-306 (A + B)

Full Marks : 50

Time : 2 hours

Answer **all** questions

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

MTM-306 A

(*Dynamical Meteorology*-I)

1. Answer any *four* questions out of eight questions : 2×4

(*a*) What is the Clapeyron diagram ? Give the characteristics of this diagram.

(*b*) Show that the pressure tendency at the earth surface becomes zero when the air motion at all levels in the atmosphere is geostrophic.

(*c*) What is the concept of Pseudo-adiabatic process ?

(*d*) The surface pressure decreases by 0·3 kPa/ 180 km in the eastward direction. A ship steaming eastward at 10 km/h measures a pressure fall of 0·1 kPa/3h. What is the pressure change on an island that the ship is passing ?

(*e*) An air parcel that has a temperature of 20 °C at the 1000 mb level is lifted dry adiabatically. What is the density when it reaches the 500 mb level ?

(*f*) Derive the amount of heat to be required to transfer for unit mass of air parcel during the isobaric process.

(*g*) What do you mean by barotropic and baroclinic atmosphere ?

(*Continued*)

(*h*) Find the expression of total water vapor content in air column in the atmosphere.

2. Answer any *four* questions out of eight questions : 4×4

(*a*) Derive the approximate horizontal momentum equation in isobaric coordinates.

(*b*) Deduce the equation of state for moist air in the following form

$$p\alpha = \frac{R^*}{m_d}\left(\frac{1+\frac{w}{\in}}{1+w}\right)T$$

where w be the mixing ratio of the moist air.

(*c*) Show that the geostrophic wind have vertical shear in the presence of horizontal temperature gradient.

(*d*) Show that the equivalent temperature (T_e) of an air parcel in the atmosphere can be obtained by the following :

$$T_e = T \exp\left(\frac{Lw_s}{C_p T}\right)$$

where symbols have usual meanings.

(*e*) What do you mean by isobaric cooling ? Show that the relative increase in dew-point temperature is about 5% of the sum of relative increase in mixing ratio and pressure.

(*f*) Derive an expression for the density ρ of an air parcel at pressure p if it is adiabatically expands from a level where pressure and density are p_s and ρ_s respectively.

(*g*) Derive the expression of the pressure gradient force in the atmosphere.

(*h*) How can Emagram be used to evaluate the change of energy ?

3. Answer any *two* questions out of four questions : 8×2

(*a*) Define homogeneous atmosphere. Show that a homogeneous atmosphere has a finite

height that depends only on the temperature at lower boundary. Compute the height of a homogeneous atmosphere with surface temperature $T_0 = 273$ K and surface pressure 100 kPa. Also prove that the pressure at the top of the homogeneous isothermal atmosphere is equal to $\frac{1}{e}$ times of the pressure at the sea level. 1 + 3 + 1 + 3

(*b*) Derive the equation of motion of an air parcel moving vertically upwards adiabatically and hence discuss stability analysis of the air parcel in the atmosphere by Parcel Method. 8

(*c*) Find the rate of change circulation in the atmosphere and interpret each term. Suppose that the air within a circular region of radius 100 km centered at the equator is initially motionless with respect to earth. If this circular air mass were moved to the north pole along an isobaric surface preserving its area, what is the amount of circulation about the circumference and the mean tangential velocity at the radius $r = 100$ km ? 6 + 2

(*d*) What do you mean by inertial and non-inertial frame of references ? Derive the relation between these two frame of references. Hence find the momentum equations of an air parcel in the Cartesian co-ordinate system in the atmosphere. 1 + 2 + 5

[*Internal Assessment* : 10 Marks]

MTM-306 B

(*Operational Research Modelling*-I)

Calculator may be used

1. Answer any *four* questions of the following : 2 × 4

(*a*) What do you mean by replacement management ?

(*b*) What are the advantages to use the simulation to solve a problem ?

(*c*) What do you mean by supply chain management (SCM) ? What is the main objective of SCM ?

(*d*) Define 'critical path' and 'critical activities'.

(*e*) State Bellman's principle of optimality.

(*f*) Explain the terms 'optimistic time' and 'most likely time' in PERT network.

(*g*) Write the distinction between deterministic and probabilistic inventory models.

(*h*) State Mortality theorem related to replacement management.

2. Answer any *four* questions of the following : 4 × 4

(*a*) Derive the conditions that determine the optimal time period of replacing an item whose maintenance cost increases with time (discrete quantity) but money value is unchanged. 4

(*b*) The cost of a new machine is Rs. 5000.00. The maintenance cost of *n*th year is given by

$c(n) = 500(n-1)$; $n = 1, 2, 3, \ldots$. Suppose that the discount rate per year is 0.05, after how many years will it be economical to replace the machine by a new one ? 4

(*c*) Explain different components of supply chain management. 4

(*d*) A baking company sells cake by its weight in kg. It makes a profit of Rs. 5.00 on every kg sold on the day it is baked. It disposes of all cakes not sold on the date they are baked, at a loss of Rs. 1.20 per kg. If the demand is known to be rectangular distribution between 2000 and 3000 kgs, determine the optimal daily amount baked. 4

(*e*) Describe a method for generating random numbers. 4

(*f*) Obtain the functional equation for solving the following the problem by dynamic programming problem :

Maximize $z = g_1(x_1) + g_2(x_2) + \ldots + g_n(x_n)$
Subject to $x_1 + x_2 + \ldots + x_n = c$
$x_1, x_2, \ldots, x_n \geq 0$. 4

(*g*) Write down the rules to construct a network. Also, describe the process to numbering the events in network analysis. 4

(*h*) When simulation method is used to solve a problem ? Describe a simulation based method to find the probability of getting head in tossing a coin. 2 + 2

3. Answer any *two* questions of the following : 8 × 2

(*a*) Based on yours assumptions, formulate and solve a dynamic inventory model with linear demand function. Hence find the optimal order quantity when demand is constant.

(*b*) A team of software developers at Microsoft is planned to rise to a strength of 50 persons and then to remain at that level. Consider the following data :

Year	1	2	3	4	5	6	7	8	9	10
Total % who have left upto the end of the year	5	30	50	60	70	75	80	85	90	100

On the basis of above information, determine

(*i*) What is the recruitment per year necessary to maintain the strength ?

(*ii*) There are 8 senior posts for which the length of service is the main criterion. What is the average length of service after which a new entrant can expect his promotion to one of these post.

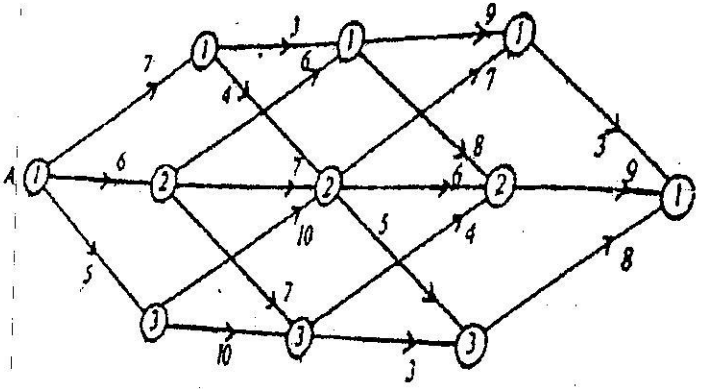
(*c*) A small project consist of seven activities, the details of which are given below :

Activity	Time estimates			Predecessor
	t_0	t_m	t_p	
A	3	6	9	None
B	2	5	8	None
C	2	4	6	A
D	2	3	10	B
E	1	3	11	B
F	4	6	8	C, D
G	1	5	15	E

(*Continued*)

Find the critical path. What is the probability that the project will be completely by 18 weeks ?

(*d*) Use dynamic programming method to find the shortest path from the vertex *A* to the vertex *B* along the edges joining various vertices lying between *A* and *B* shown in the following figure. The length of each edge is given with each edge.



[*Internal Assessment* : 10 Marks]