PG/IIIS/MTM-305/19

PG 3rd Semester Examination, 2019

MTM

PAPER —MTM-305 (A & B)

Full Marks : 40

Time : 2 hours

Answer **all** questions

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

Write the answers to Questions of each Group in separate books

MTM — 305A

(*Dynamical Oceanology*)

1. Answer any *four* questions from the following : 2 × 4

   (*a*) Write the z-component of Reynold Averaged Navier-Stokes (RANS) equation.

(*b*) Derive the expression for geopotential distance between two levels z1 and z2 in terms of standard geopotential distance and geopotential anomaly.

(*c*) Write the governing equations of motion in oceanography and indicate terms involved in the equations.

(*d*) Define rectilinear vortex.

(*e*) For the inertial motion with V_H as magnitude of velocity, find the value of Rossby number.

(*f*) Write all the averaging rules of Reynolds operator.

(*g*) Show directions of rotation around low-and high-pressure regions in northern and southern hemispheres.

(*h*) For the horizontal equations of motion when friction is included, schematically show how Coriolis, Friction and Pressure forces are related.

2. Answer any *four* questions out of eight questions : 4×4

(*a*) Considering the λ-component of Reynolds equation, define molecular stress as well as the turbulent viscosities with the similarity of molecular stress. Are the Reynolds stress preserved sysmetry ? Justify. 4

(*b*) Derive the hydrostatic equation and hence estimate the pressure experienced by a fish at a depth of 80 m. 4

(*c*) For typical horizontal length scale (L) of 800 km, horizontal speeds(U) are of the order of $0.15 ms^{-1}$ and a vertical scale length(H) of 700 m, estimate a typical vertical speed(W). 4

(*d*) When an infinite liquid contains two parallel, equal and opposite rectilinear vortices at a distance 2b, prove that the stream lines relative to this system are given by the equation

$$\log \frac{x^2 + (y-b)^2}{x^2 + (y+b)^2} + \frac{y}{b} = C,$$

the origin being the midpoint of the line joining the two vortices, taken as the y-axis. 4

(*e*) (*i*) With the help of the x- and y-momentum equations of two dimensional motion for incompressible, viscous and laminar flow, derive the equation of vorticity for this flow.

(*ii*) Also write the physical interpretation of each terms of this equation. 3 + 1

(*f*) Derive the Sverdrup equation for the Wind-driven circulation. 4

(g) Calculate the wind-driven circulation speed V_0 and Ekman depth D_E for the case of wind speed $W = 30$ m/s at latitude 45° and 80°. 4

(*h*) State and prove the Kelvin's theorem for a barotropic fluid. 4

3. Answer any *two* questions out of the following : 8 × 2

(*a*) (*i*) Derive the pressure term in the equation of motion for oceanology.

(*ii*) For the ocean with horizontal and vertical length scales 10^3 KM and 1KM, respectively and horizontal speed of order 0.1 m/s, scale all the above equations written in part-(a) and reduces to approximated equations with order of accuracy 1%. 2 + 6

(*b*) Derive the Reynolds equation for the y-component of velocity. 8

(*c*) (*i*) Define Karman Vortex by showing their positions.

(*ii*) Find the complex potential of the above Karman Vortex arrangement.

(*iii*) Find the velocity of lower rows of vortices of the above arrangement. 2 + 3 + 3

(*d*) Derive the horizontal equations of motion for current with friction (Wind-driven circulation) in terms of eddy viscosity. 8

[*Internal Assessment* : 10 Marks]

MTM – 305B

(*Advanced Optimization and Operations Research*)

1. Answer any *four* questions of the following : 2×4

   (*a*) Define integer programming problem. Give a real example of it.

   (*b*) Mention the main differences between Fibonacci method and Golden section method.

   (*c*) Define quadratically convergent method and conjugate directions of a symmetric matrix.

   (*d*) Explain different types of achievements in goal programming problem.

   (*e*) In steepest descent method, explain the name "steepest descent".

(f) Using algebraic approach show that the expression

$$ax + \frac{b}{x} + c; \; a, b > 0$$

has minimum value $2\sqrt{ab} + c$ at $x = \sqrt{b/a}$.

(g) "Revised simplex method is better than the original simplex method", why ?

(h) In a LPP if we change the objective function how does the optimal solution change ?

2. Answer any *four* questions of the following : 4 × 4

(a) Write the procedure of Fibonacci method to solve a unimodel optimization problem.

(b) Find the 1st Gomory's constraint of the following integer programming problem

Maximize $Z = 3x_1 - 2x_2$

Subject to

$$12x_1 + 7x_2 \leq 28$$

$x_1, x_2 \geq 0$ and are integers.

(*c*) Write the procedure to calculate $\hat{B}^{-1}$ in revised simplex method.

(*d*) The production manager facts the problem of job allocation among three of his teams. The processing rates of three teams are 5, 6 and 8 units per hour respectively. The normal working hours for each team are 8 hours per day. The production manager has the following goals for the next day in order of priority :

(*i*) The manager wants to avoid any under-achievement of production level, which is set at 180 units of production.

(*ii*) Any overtime operation of team 2 beyond 2 hrs and team 3 beyond 3 hrs should be avoided.

(*iii*) Minimize the sum of overtime.

For fulate this problem as a goal programming problem.

(*e*) The optimal table of the LPP

$$\text{Maximize} \quad Z = 3x_1 + 5x_2 + 4x_3$$

$$\text{subject to} \quad 2x_1 + 3x_2 \leq 8$$

$$2x_2 + 5x_3 \leq 10$$

$$3x_1 + 2x_2 + 4x_3 \leq 15$$

$$x_1, x_2, x_3 \geq 0$$

x_B	c_B		a_1	a_2	a_3	a_4	a_5	a_6
x_2	5	50/41	0	1	0	15/41	8/41	−10/41
x_3	4	62/41	0	0	1	−6/41	5/41	4/41
x_1	3	89/41	1	0	0	−2/41	−12/41	15/41

Find the set of values of c_1 and c_3 when both are changed together to maintain the same optimal solution.

(*f*) When required an artificial constraint method to solve an LPP. Explain it with an example.

(*g*) Using Fibonacci method maximise the function

$$f(x)=\begin{cases} 4x-4, & 1\le x\le 2 \\ 8-2x, & 2<x\le 4 \end{cases}$$

in the interval [1, 4], taking $n = 4$.

(*h*) Write the steps of Davidon-Fletcher-Powell method to solve on a non-linear optimization problem.

3. Answer any *two* questions from the following : 8 × 2

(*a*) Use modified simplex method to solve the goal programming problem

Minimize $Z = P_1(2d_2^- + d_3^-) + P_2 d_1^- + P_3 d_1^+$

Subject to the constraints

$$x_1 + d_1^- - d_1^+ = 450$$

$$x_2 + d_2^- - d_2^+ = 600$$

$$x_1 + x_2 + d_3^- - d_3^+ = 800$$

and $x_1, x_2, d_i^+, d_i^- \geq 0,\ i = 1, 2, 3$

(*b*) Solve by Revised simplex Method the LPP

Maximize $Z = x_1 + 2x_2$

subject to $x_1 + 2x_2 \leq 5$

$3x_1 + x_2 \leq 6$

$x_1 + x_2 \leq 3$

$x_1, x_2 \geq 0$

(*c*) Using cutting plane method, solve

Maximize $f = 7 - 2x_1 - 4x_2$

subject to $(x_1 - 4)^2 + 2(x_2 - 3)^2 - 12 \leq 0$

$x_1 + 2x_2 - 6 \leq 0$

$1 \leq x_1, x_2 \leq 6$

with the tolerance $\in = 0{\cdot}03$.

(*d*) Solve the following integer programming problem using branch and bound method.

Maximize $Z = 3x_1 + 4x_2$

subject to $2x_1 + 4x_2 \leq 7$

$5x_1 + 3x_2 \leq 15$

$x_1, x_2 \geq 0$

x_1, x_2 are integers.

[*Internal Assessment* : 10 Marks]