PG/IIIS/MTM-304/19(CBCS)

# M.Sc. 3rd Semester Examination, 2019

## MATHEMATICS

( *Discrete Mathematics* )

PAPER—MTM-304

*Full Marks* : 50

*Time* : 2 hours

Answer **all** questions

*The figures in the right-hand margin indicate marks*

*Candidates are required to give their answers in their own words as far as practicable*

*Illustrate the answers wherever necessary*

1. Answer any *four* questions : 2 × 4

   (*a*) Define the terms eccentricity and center in a tree.

(*b*) Prove that if in a graph $G$ there is one and only one path between every pair of vertices, $G$ is a tree.

(*c*) Define generating function and find a closed form for the generating function of the sequence 1, 2, 3, 4, .....

(*d*) Define language with an example.

(*e*) Prove the absorption law $a + (a \cdot b) = a$ in a Boolean algebra, $B$ for all $a, b \in B$.

(*f*) Explain spanning tree of a connected graph $G$.

(*g*) Write the duality principle in Boolean algebra. Find the dual of the Boolean expression $xy\,(y + y'z) + x'z$.

(*h*) Define chain and antichain with example.

**2.** Answer any *four* questions : 4 × 4

(*a*) Define degree of a vertex. Prove that in any graph the number of vertices of odd degree is even.

(*b*) Explain binary tree. Find the number of pendant vertices in a binary tree with $n$ vertices.

(*c*) State and prove De-Morgan's law in a Boolean algebra.

(*d*) Define poset show that the set $Z^+$ of all positive integers under divisibility relation forms a poset.

(*e*) Convert the Boolean function

$$f(x,y,z)=(x'+y+z')(x'+y+z)(x+y'+z)$$

in disjunctive normal form.

(*f*) Show that every tree has either one or two centre.

(*g*) Using mathematical induction, show that

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+.....+\frac{1}{n(n+1)}=\frac{n}{n+1}.$$

(*h*) Define finite-state machine (FSM). Let $M$ be the FSM with state table appearing as

| | $f$ | | | $g$ | | |
|---|---|---|---|---|---|---|
| $\Sigma$ / $S$ | $a$ | $b$ | $c$ | $a$ | $b$ | $c$ |
| $s_0$ | $s_0$ | $s_1$ | $s_2$ | 0 | 1 | 0 |
| $s_1$ | $s_1$ | $s_1$ | $s_0$ | 1 | 1 | 1 |
| $s_2$ | $s_2$ | $s_1$ | $s_0$ | 1 | 0 | 0 |

(*i*) Find the input set $\Sigma$, the state set $S$, the output set $O$, and initial state of $M$.

(*ii*) Draw the state diagram of $M$.

(*iii*) Find the output string for the input string *aabbcc*.

**3.** Answer any *two* questions : 8 × 2

(*a*) Define planar graph. Prove that a connected planar graph with $n$ vertices, $e$ edges has $(e - n + 2)$ faces. Also, show that a simple connected planar graph has at least one vertex of degree 5 or less.



( *Continued* )

(*b*) State the principle of inclusion-exclusion. Use the principle of inclusion-exclusion, to find the number of primes less than 100.

(*c*) Define phrase-structure grammar. Describe the classification scheme of phrase-structure grammar introduced by Noam Chomsky.

(*d*) Determine the generating function of the following sequences :

(*i*) $a_r = (x + 1)3^r$

(*ii*) $a_r = 5^r + (-1)^r 3^r + 8^r + {}^3C_r$.

[ *Internal Assessment* : 10 Marks ]