M.Sc. 3rd Semester Examination, 2019

MATHEMATICS

(*Transforms and Integral Equations*)

PAPER —MTM-302

Full Marks : 50

Time : 2 hours

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

1. Answer any *four* questions : 2 × 4

(*a*) Define exponential order on Laplace transform ? Find the exponential order on the function e^{t^n} $(n > 1)$ (if exists).

(*b*) Define the term convolution on Laplace transform.

(Turn Over)

(*c*) Define the inversion formula for Fourier transform of the function $f(x)$. What happens if $f(x)$ is continuous ?

(*d*) Define eigenvalue and eigenfunction concerning on integral equation.

(*e*) Define degenerate kernel with an example.

(*f*) Verify the final value theorem in connection with Laplace transform for the function t^3e^{-t}.

(*g*) Define the wavelet function and analyze the parameters involving in it.

(*h*) Find the Laplace transform of $f(t) = t^n$, $n > -1$.

2. Answer any *four* questions : 4 × 4

(*a*) Form an integral equation corresponding to the differential equation

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

with the initial conditions $y(0) = 1, y'(0) = 0$. 4

(*g*) Discuss the solution procedure for solving the homogeneous Fredholm integral equation with separable kernel. 4

(*h*) Find the Fourier Cosine transform of e^{-at^2}. 4

3. Answer any *two* questions : 8 × 2

(*a*) (*i*) Solve the integral equation

$$y(x) = x + \lambda \int_{-\pi}^{\pi} [x\cos(t) + t^2 \sin(x) + \cos(x)\sin(t)]y(t)dt\,.$$ 5

(*ii*) Evaluate :

$$\left\{ \int_0^t J_0(s) J_1(t-s) ds \right\}$$ 3

(*b*) Find the solution of the following problem of free vibration of a stretched string of infinite length PDE :

$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = 0, \;\; -\infty < x < \infty,$$

(*b*) State and prove convolution theorem concerning on Fourier transform. 4

(*c*) If $L\{f(t)\} = F(p)$ which exists $Real(p) > \gamma$ and $H(t)$ is unit step function, then prove that for any α, $L\{H(t-\alpha)f(t-\alpha)\} = e^{-p\alpha}F(p)$ which exists for $Real(p) > \gamma$. 4

(*d*) If the function $f(t)$ has the period $T > 0$, then prove that

$$L\{f(t)\} = \frac{1}{1-e^{-pT}} \int_0^T f(t)e^{-pt}\,dt.$$ 4

(*e*) Solve the following ODE by Laplace transform technique

$$y''(t) + a^2 y(t) = f(t)$$

with initial conditions $y(0) = 1$, $y'(0) = -1$. 4

(*f*) Define the continuous wavelet function and also explain the inverse wavelet transform. Write some important applications of wavelets. 1 + 1 + 2 with boundary conditions $u(x, 0) = f(x)$,

$-\infty < x < \infty$, $\dfrac{\partial u(x,0)}{\partial t} = g(x)$, u and $\dfrac{\partial u}{\partial x}$ both

vanish as $|x| \to \infty$. 8

(c) (i) Find the Laplace transform of $J_0(t)$ by using initial value theorem. 5

(ii) Solve the integral equation

$$y(x) = \sin(x) + 2\int_0^x \cos(x-t)y(t)dt.$$ 3

(d) (i) Find the characteristic numbers and eigen functions of the homogeneous integral equation

$$y(x) = \lambda \int_0^{\pi} K(x,t)y(t)dt,$$

where $K(x,t) = \begin{cases} \cos(x)\sin(t), & 0 \le x \le t \\ \cos(t)\sin(x), & t \le x \le \pi \end{cases}$ 6 (*ii*) Find the Fourier transform of

$$f(x) = e^{-a|x|} a > 0.$$ 2

[*Internal Assessment* : 10 Marks]