M.Sc. 1st Semester Examination, 2019

MATHEMATICS

PAPER –MTM-106

Full Marks : 25

Time : 1 hours

Answer **all** questions

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

1. Answer any *two* questions : 2 × 2

   (*a*) Describe spanning tree of a connected graph G.

   (*b*) Prove that there is one and only one path between every pair of vertices in a tree T.

(*c*) Define isomorphic graph and give an example.

(*d*) Find the digraph whose adjacency matrix is

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 0 & 0 \end{pmatrix}.$$

2. Answer any *two* questions : 4 × 2

(*a*) Define circuit of a graph G and prove that a circuit free graph with n vertices and $(n-1)$ edges is a tree.

(*b*) Explain fundamental cut-set. With respect to a given spanning tree T, show that each branch determines a fundamental cut-set S, which is contained in every fundamental circuit associated with the chords in S and in no others.

(*c*) Show that a simple connected planar graph with 6 vertices and 12 edges, each of the face is bounded by 3 edges.

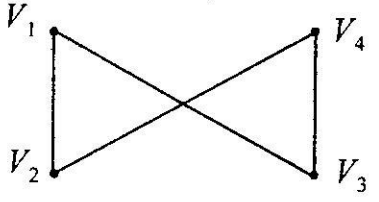
(*d*) Prove that the graph $K_{3,3}$ and K_5 are not planar.

3. Answer any *one* question : 8×1

(*a*) (*i*) State and prove Euler's theorem for a connected planar graph. 4

(*ii*) Show that a simple graph with n vertices and m components can have atmost $\frac{1}{2}(n-m)(n-m+1)$ edges. 4

(*b*) Describe four-colour problem in graph theory. Consider the graph shown in figure, find the number of walks of length three from V_2 to V_4 and also check the connectedness of the graph. 3 + 5



[*Internal Assessment* : 5 Marks]