M.Sc. 1st Semester Examination, 2019

MATHEMATICS

(Classical mechanics and Non-linear Dynamics)

PAPER —MTM-105

Full Marks : 50

Time : 2 hours

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

Notations and symbols have their usual meanings

1. Answer any *four* questions : 2×4

   (*a*) What do you mean by generalized coordinates ? How do they differ from conventional coordinates ?

(*b*) What do you mean by bifurcation of a system ?

(*c*) Show that the distribution law over addition is true for Poisson bracket.

(*d*) Show that for conservative Holonomic system,

$$\frac{\partial L}{\partial \dot{q}_j} = \int \frac{\partial L}{\partial q_j}\, dt.$$

(*e*) Show that the canonical transformation of a dynamical system can be determined if the generating function is given.

(*f*) State the principle of least action. Write down it's importance.

(*g*) What do you mean by bifurcation in non-linear dynamics.

(*h*) A particle of mass *m* moves in a plane. Write down the Lagrangian for this particle using plane polar coordinates.

2. Answer any *four* questions : 4 × 4

(*a*) Show that the Poisson bracket is invariant under canonical transformation. 4

(*b*) What is the effect of the Coriolis force on a particle falling freely under the action of gravity. 4

(*c*) The Lagrangian for a coupled harmonic oscillator is given by

$$L=\frac{1}{2}\left(\dot{q}_1^2+\dot{q}_2^2\right)-\frac{1}{2}\left(w_1^2q_1^2+w_2^2q_2^2\right)+\alpha q_1q_2,$$

where α, w_1, w_2 are constants and q_1, q_2 are suitable coordinates. Find the Hamiltonian of the system. Write down the Lagrange's equation of motion. 4

(*d*) Deduce the Lagrange equation of motion for conservative and holonomic system. 4

(*e*) If a transformation from p, q to P, Q be canonical then show that

$$\sum_i\left(\delta p_i dq_i-\delta q_i dp_i\right)$$

is invariant. 4

(*f*) State and prove Jacobi identity in connection with canonical transformation. 4

(*g*) (*i*) Derive Hamilton's equations of motion in terms of Poisson bracket.

(*ii*) If H is the Hamiltonian and f is any function depending on position, momentum and time show that

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + [f, H].$$ 2 + 2

(*h*) Find the equations of motion of a rigid body rotating with an angular velocity w about a fixed point. 4

3. Answer any *two* questions : 8 × 2

(*a*) Consider the equilibrium configuration of the molecule such that two of its atoms, each of mass M, are symmetrically placed on each side of the third atom of mass m. All three atoms are collinear. Assume that the motion is along the line of molecules and there being no interaction between the end atoms. Compute the kinetic energy and potential energy of the system and discuss the motion of the atoms. 8

(*b*) (*i*) Let

$$J = \int_{x_0}^{x_1} F(y_1, y_2, y_3, ..., y_n, y_1', y_2', y_3', ..., y_n', x)dx,$$

where $y_1, y_2, ..., y_n$ are unknown functions of x. Derive a differential equations to find the curve $y_i = y_i(x)$, $i = 1, 2, ..., n$ which will optimize J. 5

(*ii*) Determine the path for which the functional

$$\int_{-1}^{1}\left(\frac{1}{2}ay''^2 + by\right)dx$$

subject to $y(-l) = 0$, $y'(-l) = 0$, $y(l) = 0$, $y'(l) = 0$ is extremum. 3

(*c*) (*i*) In special theory of relativity, show that

$$m = \frac{m_0}{\sqrt{1 - v^2 / c^2}}.$$ 5

(*ii*) Calculate the length and the direction of a rod of length 500 cm in a frame of reference which is moving with a velocity equal to 0.8 c, in a direction making an angle of 45° with the rod. 3

(*d*) Solve the following dynamical system

$$\dot{x}_1 = -x_1 - 3x_2, \;\; \dot{x}_2 = 2x_2.$$

Also, sketch the phase portrait. 8

[*Internal Assessment* : 10 Marks]