2019

MSc

2nd Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER – MTM-201

Full Marks: 50 marks

Time: 2 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

(FLUID MECHANICS)

1. Answer any ***FOUR*** questions out of ***EIGHT*** questions: 4x2 = 8

   a) The velocity components for two-dimensional incompressible steady flow are given as $u = 3x^2 - 2xy$ and $v = y^2 - 6xy$. Show that such a motion is possible.

   b) Draw an infinitesimally small moving element and show all the energy fluxes along x-direction associated with the above element.

   c) Distinguish between laminar and turbulent flows.

   d) Assuming the velocity profile

   $\frac{u}{u_\infty} = 2\frac{y}{\delta} - (\frac{y}{\delta})^2$, determine the energy thickness.

   e) Approximate the boundary layer equations outside the boundary layer.

   f) Arrange the velocities (x- and y- components) and pressure in the Staggered Gird and then write the advantage – disadvantage of this arrangement.

   g) Show the geometric interpretation of forward, backward and central differences for $\frac{dy}{dx}$ graphically as a slope.

   h) Using the general technique, derive the expression for $\frac{dy}{dx}$ at $x=x_i$ in terms of y_i, y_{i+1} & y_{i+2}.

2. Answer any ***FOUR*** questions out of ***EIGHT*** questions: 4 x 4 = 16

   a) Derive Euler's equations of motion for a two-dimensional flow in the presence of external body force.

   b) Write down all the possible boundary conditions for tangential and normal component of velocity and temperature.

c) The velocity distribution for fluid flow over a flat plate is given by $u = \frac{1}{2}y - y^{3/2}$, then determine the shear stress at a point where the velocity is maximum.

d) What do you mean by analytical/exact solution of Navier-Stokes Equation? With the necessary assumptions, find the exact solution for the case of Couette flow.

e) Write down the complete Navier-Stokes equations in conservation form for compressible flow, and then reduce these equations for incompressible flow. Finally write down these equations, for later case, in vector form.

f) Draw the flow chart of solution process of finite difference method (FDM) of an unsteady flow problem.

g) Discretize the one dimensional transport equation $\frac{\partial T}{\partial t} + \alpha \frac{\partial T}{\partial x} = \alpha \frac{\partial^2 T}{\partial t^2}$, where a and α are constants, using Crank-Nicolson scheme and hence write the algebraic expression in a matrix form for the case of Neumann boundary conditions.

h) Write the algebraic formula for $\frac{dy}{dx}$ using forward, backward, central, and three points asymmetry forward as well as backward schemes. Also write the order of accuracy of these schemes.

3. Answer any ***TWO*** questions out of ***FOUR*** questions: 2x8= 16

(a) (i) What are the observations of Ludwig Prandtl for boundary layer theory?

(ii) Based on the above observations, derive the set of governing equations for the boundary layer flow along a flat plate. Write down the proper boundary conditions for the above set of equations. [2+6]

(b) (i) *Write down the x-component* of Navier-Stokes equation and energy equation for Newtonian, incompressible, viscous fluid flow with negligible gravity and radiation effects.

(ii) Make the above equations in non-dimensional form (Navier-Stokes equation in terms of Reynolds number Re = $\frac{UL}{\gamma}$, and energy equation in terms of Re and Prandtl number Pr = $\frac{\gamma}{\alpha}$) with the help of characteristics length, velocity, pressure and temperature as L, U, ρU^2 and $T_w - T_c$, respectively, where symbols have their usual meaning. [2+6]

(c) Discretize the heat conduction equation $\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial t^2}$ using FTCS (forward time central space), DuFort-Frankel and three-time fully implicit schemes followed by their leading term of truncation error and stability restriction. [8]

(d) Consider steady, laminar, fully developed flow between two parallel plates separated by a distance 2H. The fluid is driven between the plates by an applied pressure gradient in the x-direction. It is assumed that conduction in the y-direction is much greater than conduction in x-direction.

i) Determine the fully developed velocity distribution of the fluid as a function of the mean velocity.

ii) Determine the fully developed temperature distribution as a function of the surface and mean temperatures. [5+3]

[Internal Assessment: 10 Marks]