2019

MSc

4th Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER – MTM-404 (OM/OR)

Full Marks : 50

Time : 2 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Answer question no. *1* and any ***FOUR*** from the rest.

1. Answer any **FOUR** questions out of six questions (4x2)

(a) Arrange the velocities (x- and y-components) and pressure in the Staggered Gird and then write the advantage – disadvantage of this arrangement.

(b) Calculate Rossby radius of deformation for a deep ocean with height = 4000 m and a shallow ocean with height = 100m

(c) Write the coupled set of equations for the horizontal velocity components and the surface elevation of shallow water .

(d) Write the empirical formula for wave run-up over a beach or a structure slope.

(e) If the liquids are at rest, then evaluate the speed of propagation (C) for steady motion of progressive waves of an interface.

(f) What is the porosity of sediment and draw a schematic diagram of wave over mud region.

2. (a) With the notation P,E,W,N,S---, draw the control volume for u-velocity and arrange both the velocities and pressure around the control volume.

(b) Write the x-momentum equation in the form of first order derivative with suitable substitutions.

(c) Applying the finite volume method (FVM) to the above x-momentum equation and then using the central differences for the first derivative at the faces of the control volume, write the discretize algebraic expression of the above equation . (2+2+4)

3. (a) With necessary assumptions, write the set of governing equations for sverdrup waves in shallow water.

(b) Find the solution of the above equations for the surface elevation and velocity components .

(c) Show that the horizontal velocity vector describes an ellipsis where the ratio of the major axis to the minor axis is $|\omega / f|$. (2+4+2)

4. a) Calculate the circulation within a small fluid element with area $\delta x \delta y$.

b) Consider an arbitrary large fluid element, and divide it into small squares . Then calculate the circulation within the area.

(c) Discuss the conservation of potential vorticity for all three cases : in the absence of stretching, in the absence of planetary vorticity and in the absence of relative vorticity. (3+3+2)

5. (a) For steady motion of progressive waves at an interface when the upper surface is free,then derive the velocity propagation if the lower liquid is an infinite depth. (5)

(b) Let a shallow be filled with oil and water; let the depth of water be k and its density σ and depth of oil be h and its density ρ, show that, if g be the gravity and u be the velocity propagation of long waves ,

$$\frac{u^2}{g}=\frac{1}{2}(h+k)+\frac{1}{2}\{(h-k)^2+4hk\frac{\rho}{\sigma}\}^{\frac{1}{2}}$$ 3

6. (a) How to generate tsunami wave and write the mathematical expression of wave speed and time travel of tsunami. 3

(b) Discuss the properties and social impact of tsunami. 3

(c) Write the expressions for conservation of momentum and energy equation of tsunami wave. 2

7. Derive the velocity potential of water wave region and viscous mud region and plot the dispersion relationship for waves over an infinite deep water fluid. 3+4+1

MTM-404-(OR)

1. Answer any ***FIVE*** questions. 5x2 = 10

a) What is the degree of difficulty in connection with Geometric programming ?

b) Write the differences between quadratic programming and non linear programming ?

c) Define Nash equilibrium solution and Nash equilibrium out come in pure strategy for bimatrix game.

d) What is the differentiable concave function ?

e) Define Pareto optimal solution for a multi-objective non linear programming problem.

f) State Kuhn-Tucker stationary point necessary optimality theorem .

g) What is the theorem of alternatives ?

h) State Karlin's constraint qualification.

Answer any ***THREE*** questions from Q. **2** to **6** . 3x10=30

2. (a) Let X be an open set in R^n, and θ and g be differential and convex on X. Let $\bar{x}$ solves the minimization problem and g satisfy the Kuhn-Tucker constraint qualification. Show that there exists a $\bar{u} \in R^m$ such that $(\bar{x}, \bar{u})$ solves the dual maximization problem and $\theta(\bar{X}) = \Psi(\bar{X}, \bar{Y})$.

(b) Prove that all strategically equivalent bimatrix games have the same Nash equilibria.

(6+4=10)

3 (a) Let θ be a numerical differentiable function on an open convex set $\Gamma \subset R^n$. Θ is convex if and Only if $\theta(x^2) - \theta(x^1) \geqq \nabla\theta(x^1)(x^2 - x^1)$ for each $x^1, x^2 \in \Gamma$.

(b) Define the following terms :

(i) The (primal) quadratic minimization problem (QMP).

(II) The quadratic dual (maximization) problem (QDP). (6+4=10)

4. (a) State and prove Fritz- John saddle point sufficient optimality theorem . What are the Basic differences between the necessary criteria and sufficient criteria of FJSP. 6

(b) Define the following :

(i) Minimization problem ;

(ii) Local minimization problem . 4

5. (a) How do you solve the following geometric programming problem ?

Find X = $\begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix}$ that minimizes the objective function

$$f(x)=\sum_{j=1}^{n} U_j(x) = \sum_{j=1}^{N}\left(c_j \prod_{i=1}^{n} x_i^{a_{ij}}\right)$$

$c_j > 0,\ x_i > 0,\ a_{ij}$ *are real numbers,* $\forall\ i, j.$

(b) Derive the Kuhn-Tucker conditions for quadratic programming problem. (6+4 =10)

6. (a) State and prove Slater's theorem of alternative.

(b) Use the chance constraints programming techniques to find an equivalent deterministic LPP to the following stochastic programming problem.

Minimize $F(x) = \sum_{j=1}^{n} c_j x_j$

Subject to $P\left[\sum_{j=1}^{n} a_{ij} x_j \leq b_i\right] \geq p_i$

$x_j \geq 0,\ i, j = 1,2, \cdots n$

When b_i is a random variable and p_i are the specified probabilities. (3+7 =10)

(Internal Assessment – 10 Marks)