2019

MSc

4th Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER – MTM-402

Full Marks : 50

Time : 2 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

UNIT – I

Full Marks-25

(Fuzzy Sets and their Applications)

Answer Q. No. *1* and any *TWO* from the rest .

1. Answer any *TWO* questions : 2x2

a) Show by examples that law of excluded middle and law of contradiction does not hold for fuzzy sets (no proof is required)

b) Explain Bellman and Zadeh's fuzzy optimality principle with an example.

c) Is every fuzzy set a fuzzy number? Explain with examples.

2. a) Define arithmetic operations on interval numbers . 3

b) Explain why for interval number

$$[2\ ,4]-[2,4]\neq[0,0].$$ 2

c) Draw the graph of the fuzzy set $\tilde{A}$ ={ old person } whose membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x \le 50 \\ \frac{x-50}{70}, & 50 < x < 70 \\ 1, & x \geq 70. \end{cases}$$

How much old is a person of age 40, 55, 60, 80. 2+1

3. a) "Multiplication of any two triangular fuzzy numbers is a triangular fuzzy number" , Justify your answer. 5

b) For fuzzy set $\tilde{A}$, where $\mu_{\tilde{A}}(x) = \begin{cases} \left|\frac{x-5}{3}\right|, & 2 \le x \le 8 \\ 0 & ,otherwise \end{cases}$

Find i) $\tilde{A}^c$ i.e. complement of $\tilde{A}$. 3

ii) $\tilde{A}_{0.4}$ i.e. 0.4 – Cut of $\tilde{A}$.

4. a) Using Zimmermann's method , determine the crisp LPP equivalent to the fuzzy LPP

$\tilde{Max}\ z = 3x_1 + 4x_2 + 2x_3$

Subject to $x_1 - 2x_2 + 4x_3 \leq 34$ to 38

$2x_1 + 3x_2 - 5x_3 \leq 39$ to 43

$x_1, x_2, x_3 \geq 0$

Where lower bound of the value of the fuzzy objective function is 55 with tolerance 4. 4

b) Describe the process of converting fuzzy LPP with all parameters as triangular fuzzy numbers to equivalent crisp problem. 4

[Internal Assessment: 05 marks]

UNIT II

Full Marks-25

Stochastic Process and Regression

Answer question no ***1*** and any ***TWO*** from the rest.

1. Answer any ***TWO*** questions : 2x2

a) Discuss how a Markov Chain can be represented as a graph.

b) Show that the bivariate correlation coefficients r_{12}, r_{13} and r_{23} must satisfy the inequality $r_{12}^2 + r_{13}^2 + r_{23}^2 - 2r_{12}r_{13}r_{23} \leq 1$.

c) Define transient and persistent states. When a persistent state is called null-persistent?

d) State Galton – Watson branching process.

2.a) Suppose that probability of a dry day following a rainy day is $\frac{2}{3}$ and that the probability of a rainy day following a dry day is $\frac{1}{2}$ and t.p.m. P = $\begin{pmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$. If June 2 is a dry day then find the probability that June 4 and June 6 are dry day.

b)Let $\{x_n, n \geq 1\}$ be a Markov chain having state space S= $\{1, 2, 3, 4\}$ and transition Matrix

$$P = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

. Identify the states as transient, persistent, ergodic. [4+4]

3. Let $\{X_n, n \geq 0\}$ be a branching process.Show that if $m = E(X_1) = \sum_{k=0}^{\infty} kp_k$ and σ^2 = Var (X_1) then $E(X_n) = m^n$ and

$$\text{Var}(X_n) = \begin{cases} \frac{m^{n-1}(m^n - 1)}{m - 1}\sigma^2, & \text{if } m \neq 1 \\ n\sigma^2, & \text{if } m = 1 \end{cases}$$

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4. Prove that

$$r_{1.23...p} = \left(1 - \frac{|R|}{R_{11}}\right)^{\frac{1}{2}},$$

where the symbols have their usual meaning. 8

[Internal Assessment - 05]