Journal of Physical Sciences, Vol. 24, 2019, 143-152
ISSN: 2350-0352 (print), www.vidyasagar.ac.in/publication/journal
Published on 6 June 2019

Analysis of a FM/FM/1 WV and Vacation Interruptions with Set-Up Times

G.Kannadasan[1]**, *D.Devi***[2] **and *N.Sathiyamoorthi***[3]

[1]Department of Mathematics, Annamalai University
Annamalainagar-608002, India.
[3]Kamban College of Arts and Science for Women,
Department of Mathematics, Tiruvannamalai – 606603
[3]Department of Mathematics, Annamalai University
Annamalainagar-608002, India
Corresponding Author: n.satyamurthi@gmail.com

Received 2 December 2018; accepted 30 January 2019

ABSTRACT

Using fuzzy technique to analysis of a single server fuzzy queue working vacation(WV) and vacation interruptions with set-up time is discussed in this paper. We obtain model in fuzzy environment as the probability of the server is in a closed-down period, server is in a set-up period, server is in a regular busy period, membership function of the mean queue length and mean waiting time. Finally, numerical results are presented to show the effects of system parameters.

Keywords: FM/FM/1 Queue; Single server; Working vacation; Busy period; Set- up period; Close-down period; vacation interruptions.

Mathematical Subject Classification (2010): 6025K, 68M20, 90B22

1. Introduction

Working vacation is a kind of semi-vacation policy was introduced by Servi and Finn [9] a customer is served at a lower rate rather than completely stopping the service during a vacation. In the classical vacation queuing models, during the vacation period the server doesn't continue on the original work and such policy may cause the loss or dissatisfaction of the customers. For the working vacation policy, the server can still work during the vacation and may accomplish other assistant work simultaneously. So, the working vacation is more reasonable than the classical vacation in some cases. The details can be seen in the monograpraphs of Takagi [12], Tian and Zhang [13] and the survey of Doshi [3]. However, in these models, the server stops the original work in the vacation period and cannot come back to the regular busy period until the vacation period ends. Baba [1] studied a GI/M/1 queue with working vacations by using the matrix analytic method. Banik et al. [2] analyzed the GI/M/1/N queue with working vacations. Liu et al. [7] established a stochastic decomposition result in the M/M/1 queue with working vacations. In 2007, Li and Tian [8] first introduced vacation interruption policy and studied an M/M/1 queue. Majid and Manoharan [10] analyzed the M/M/1 queue with single working vacation and vacation interruption using matrix geometric method. For example, when the number of customers exceeds the special value and if the server continues to take the vacation, the cost of waiting customers and providing service in the vacation period will be large. Using the method of a supplementary variable, Zhang and Hou [14] considered an M/G/1 queue with working vacations and vacation interruption. Sreenivasan et al. [11] studied an MAP/PH/1 queue with working vacations, vacation and N-Policy.

Power-Saving in ICT systems is an important issue because ICT devices consume a large amount of energy. One simple method is to turn off on idle device and to switch it on again when some jobs arrive. This is because in the current technology idle devices still consume of peak processing a job on the other hand, a quick response in crucial for delay sensitive applications. An off server needs some setup time in order to be active during which the server consumes energy but cannot process a job. Thus, there is a trade-off can be analyzed using single server queuing models with setup times which are extensively studied in the literature. Our models are suitable for a down link of a mobile station with a power saving mode. A mobile station receives data from a base station. Arriving messages are stored in the base station and the mobile station downloads these messages from the base station upon the completion of a download, if there are no messages in the base station the mobile station is turned off in order to save energy. However, when a message arrives, the base station sends a signal in order to wake up the mobile station. The mobile station needs some random setup time to be active so as to receive waiting messages.

Kalayanaraman et al. [4] introduced a single server vacation queue with fuzzy service time and vacation time distributions with some performance measures. Kalyanaraman, et al. [5] gave a single server fuzzy queue with group arrivals and server vacation. Kannadasan and Sathiyamoorthi [6] investigate the $FM/FM/1$ queue with single working vacation.

2. The model in fuzzy environment

In this section, the arrival rate, service rate, arriving customers can be served at a mean service rate and working vacation time are assumed to be fuzzy numbers $\bar{\lambda}, \overline{\gamma_1}, \bar{\gamma}_2, \bar{\theta}$ respectively. Now

$$\bar{\lambda} = \{x, \mu_{\bar{\lambda}}(x); x \in S(\bar{\lambda})\},$$

$$\bar{\gamma}_1 = \{y_1, \mu_{\bar{b}}(y_1); y_1 \in S(\bar{\gamma}_1)\},$$

$$\bar{\gamma}_2 = \{y_2, \mu_{\bar{\theta}}(y_2); y_2 \in S(\bar{\gamma}_2)\}$$

$$\bar{\theta} = \{z, \mu_{\bar{\theta}}(z); z \in S(\bar{\theta})\}.$$

where $S(\bar{\lambda})$, $S(\bar{\gamma}_1)$, $S(\bar{\gamma}_2)$ and $S(\bar{\theta})$ are the universal set's of the arrival rate, service rate, arriving customers can be served at a mean service rate, working vacation time respectively. It define $f(x, y_1, y_2, z)$ as the system performance measure related to the above defined fuzzy queuing model, which depends on the fuzzy membership function $f(\bar{\lambda}, \bar{\gamma}_1, \bar{\gamma}_2, \bar{\theta})$. Applying Zadeh's extension principle (1978) the membership function of the performance measure $f(\bar{\lambda}, \bar{\gamma}_1, \bar{\gamma}_2, \bar{\theta})$ can be defined as

$$\mu_{\bar{f}(\bar{\lambda},\bar{\gamma}_1,\bar{\gamma}_2,\bar{\theta})}(H) = \sup_{\substack{x\in S(\bar{\lambda})\\ y_1\in S(\bar{\gamma}_1)\\ y_2\in S(\bar{\gamma}_2)\\ z\in S(\bar{\theta})}} \left\{\mu_{\bar{\lambda}}(x), \mu_{\bar{\gamma}_1}(y_1), \mu_{\bar{\gamma}_2}(y_2), \mu_{\bar{\theta}}(z) / H = f(x, y_1, y_2, z)\right\} \tag{1}$$

If the α- cuts of $f(\bar{\lambda}, \bar{\gamma}_1, \bar{\gamma}_2, \bar{\theta})$ degenerate to some fixed value, then the system performance is a crisp number, otherwise it is a fuzzy number.

We obtain the membership function of some performance measures, namely Probability of the server is in a close-down period, Probability of the server is in a set-up period, Probability of theserver is in a regular busy period, Membership function of the mean queue length, Membershipfunction of the mean waitingtime. For the system in terms of this membership function are:

$$\mu_{\overline{P_1}}(A) = \sup_{\substack{x\in S(\bar{\lambda})\\ y_1\in S(\bar{\gamma}_1)\\ y_2\in S(\bar{\gamma}_2)\\ z\in S(\bar{\theta})}} \left\{\mu_{\bar{\lambda}}(x), \mu_{\bar{\gamma}_1}(y_1), \mu_{\bar{\gamma}_2}(y_2), \frac{\mu_{\bar{\theta}}(z)}{A}\right\}, \tag{2}$$

where

$$A = \left[\frac{\left[\frac{x+z+y_2}{z+y_2} + \frac{z(x+y_2+z)}{x} + \frac{zy_1y_2(zx+z^2+zy_2+x)(z+y_2)^2}{x+z+y_2} + \frac{x^2z(x+z+y_2)y_1}{(xy_2)^2(y_1-x)} + \frac{(x^2+2xz+z^2+zy_2)y_1-x}{x+z+y_2}\right]^{-1}}{\left(\frac{x}{x^2+xy_2xz+x}\right)\left(\frac{y_1-x}{y_1}\right)\left(\frac{z+y_2}{x+z+y_2}\right)}\right]\frac{z}{x}$$

$$\mu_{\overline{P_2}}(B) = \sup_{\substack{x\in S(\bar{\lambda})\\ y_1\in S(\bar{\gamma}_1)\\ y_2\in S(\bar{\gamma}_2)\\ z\in S(\bar{\theta})}} \left\{\mu_{\bar{\lambda}}(x), \mu_{\bar{\gamma}_1}(y_1), \mu_{\bar{\gamma}_2}(y_2), \mu_{\bar{\theta}}(z) / B\right\} \tag{3}$$

where

$$B = \left[\frac{\left[\frac{x+z+y_2}{z+y_2} + \frac{z(x+y_2+z)}{x} + \frac{zy_1y_2(zx+z^2+zy_2+x)(z+y_2)^2}{x+z+y_2} + \frac{x^2z(x+z+y_2)y_1}{(xy_2)^2(y_1-x)} + \frac{(x^2+2xz+z^2+zy_2)y_1-x}{x+z+y_2}\right]^{-1}}{\left(\frac{x}{x^2+xy_2xz+x}\right)\left(\frac{y_1-x}{y_1}\right)\left(\frac{z+y_2}{x+z+y_2}\right)}\right]\left(\frac{\theta(\lambda+\gamma_2+\theta)}{\lambda}\right)$$

$$\mu_{\overline{P_3}}(C) = \sup_{\substack{x\in S(\bar{\lambda})\\ y_1\in S(\bar{\gamma}_1)\\ y_2\in S(\bar{\gamma}_2)\\ z\in S(\bar{\theta})}} \{\mu_{\bar{\lambda}}(x), \mu_{\bar{\gamma}_1}(y_1), \mu_{\bar{\gamma}_2}(y_2), \mu_{\bar{\theta}}(z)/C\}, \qquad (4)$$

where,

$$C = \left[\frac{x+z+y_2}{z+y_2} + \frac{z(x+y_2+z)}{x} + \frac{zy_1y_2(zx+z^2+zy_2+x)(z+y_2)^2}{x+z+y_2} + \frac{x^2z(x+z+y_2)y_1}{(xy_2)^2(y_1-x)} + \frac{(x^2+2xz+z^2+zy_2)y_1-x}{x+z+y_2}\right]^{-1}$$

$$\left[\begin{array}{c}\left(\frac{xy_2(xz+z^2+zy_2+x)(y_1y_2+y_1z)}{x+z+y_2}\right)\\ \left(\frac{xy_1}{(z+y_2)(y_1-x)}\right)+\left(\frac{z(x+y_2+z)}{x^2xy_2+xz+x}\right)\left(\frac{x^3+x^2+x^2y_2+x^2z}{xy_1(x+z+y_2)}\right)\left(\frac{(x^2+xy_2+xz)xy_1}{y_1-x}\right)\\ +\left(\frac{x^2+z^2+3xz+xy_2+zy_2}{y_1(x+z+y_2)}\right)\left(\frac{y_1}{y_1-x}\right)\end{array}\right]$$

$$\mu_{\overline{E(L)}}(D) = \sup_{\substack{x\in S(\bar{\lambda})\\ y_1\in S(\bar{\gamma}_1)\\ y_2\in S(\bar{\gamma}_2)\\ z\in S(\bar{\theta})}} \{\mu_{\bar{\lambda}}(x), \mu_{\bar{\gamma}_1}(y_1), \mu_{\bar{\gamma}_2}(y_2), \mu_{\bar{\theta}}(z)/D\}, \qquad (5)$$

where

$$D = \left[\frac{\left[\frac{x+z+y_2}{z+y_2} + \frac{z(x+y_2+z)}{x} + \frac{zy_1y_2(zx+z^2+zy_2+x)(z+y_2)^2}{x+z+y_2} + \frac{x^2z(x+z+y_2)y_1}{(xy_2)^2(y_1-x)} + \frac{(x^2+2xz+z^2+zy_2)y_1-x}{x+z+y_2}\right]^{-1}}{\left(\frac{x}{x^2+xy_2xz+x}\right)\left(\frac{y_1-x}{y_1}\right)\left(\frac{z+y_2}{x+z+y_2}\right)}\right]$$

$$\left[\begin{array}{l}\left[\frac{x-xy_2-x^2-xz}{y_1(x+z+y_2)}+\frac{z}{x}\left(\frac{x^2y_1+xzy_1+xy_1y_2-x^3-x^2z-x^2y_2+x}{y_1(x^2+xz+xy_2+x)}\right)\right. \\ \left.\qquad +\frac{x^2+z^2+2xz+zy_2}{y_1(x+z+y_2)}\left(\frac{x}{x^2+xy_2+xz+x}\right)\left(\frac{z+y_2}{x+z+y_2}\right)\right] \\ +\left[\left(\frac{(2x-x^2)(z+y_2)}{(x+z+y_2)^2}\right)\left(\begin{array}{c}\frac{xy_1-xy_2-x^2-xz}{y_1(x+z+y_2)} \\ +\frac{xy_2(xz+z^2+zy_2+x)}{(x+z+y_2)^2}\end{array}\right)\left(\frac{x}{x^2+xy_2+xz+x}\right)\right] \\ +\left[\begin{array}{c}\left(\frac{2(x+z+y_2)(y_2+z+2)^2-(x+z+y_2)^2(x+y_2+z+1)}{(y_2+z+2)^2}\right) \\ \left[\frac{xz+z^2+zy_2}{x^2+xz+xy_2+x}-\frac{z}{y_1}+\frac{xzy_1}{x^2+xz+xy_2+x(y_1{}^2-xy_1)}\right]\left(\frac{z+y_2}{x+z+y_2}\right)\end{array}\right]\end{array}\right]$$

$$\mu_{\overline{E(W)}}(E)=\sup_{\substack{x\in S(\overline{\lambda})\\ y_1\in S(\overline{\gamma}_1)\\ y_2\in S(\overline{\gamma}_2)\\ z\in S(\overline{\theta})}}\left\{\mu_{\overline{\lambda}}(x),\mu_{\overline{\gamma}_1}(y_1),\mu_{\overline{\gamma}_2}(y_2),\frac{\mu_{\overline{\theta}}(z)}{E}\right\}, \tag{6}$$

where

$$E=\left[\frac{\left[\begin{array}{c}\frac{x+z+y_2}{z+y_2}+\frac{z(x+y_2+z)}{x}+\frac{zy_1y_2(zx+z^2+zy_2+x)(z+y_2)^2}{x+z+y_2} \\ +\frac{x^2z(x+z+y_2)y_1}{(xy_2)^2(y_1-x)}+\frac{(x^2+2xz+z^2+zy_2)y_1-x}{x+z+y_2}\end{array}\right]^{-1}}{\left(\frac{x}{x^2+xy_2xz+x}\right)\left(\frac{y_1-x}{y_1}\right)\left(\frac{z+y_2}{x+z+y_2}\right)}\right]$$

$$\left[\begin{array}{l}\quad\left(\frac{x(y_1-y_2-x-z)}{y_1(x+z+y_2)}+\frac{xy_2(xz+z^2+zy_2+x)}{(x+z+y_1)^2}\right)\left(\frac{x}{x^2+xy_2+xz+x}\right) \\ +\left[\left(\frac{xz+z^2+zy_2}{x^2+xz+xy_2+x}\right)\left(\frac{x}{y_1}\right)\left(\frac{xzy_1}{x^2+xz+xy_2+x(y_1{}^2-xy_1)}\right)\right]\left(\frac{zy_2}{x+z+y_2}\right) \\ \qquad +\left(\frac{1}{y_1-x}\right)\end{array}\right]$$

Using the fuzzy analysis technique explain, we can find the membership of $\overline{P_1},\overline{P_2},\overline{P_3},\overline{E(L)},\overline{E(W)}$ as a function of the parameter α. Thus the α-cut approach can be used to develop the membership function of $\overline{P_1},\overline{P_2},\overline{P_3},\overline{E(L)},\overline{E(W)}$.

3. Performance of measure

The following performance measure are studied for this model in fuzzy environment.

Probability of the server is in a close-down period

Based on Zadeh's extension principle $\mu_{\overline{P_1}}(A)$ is the superimum of minimum over $\{\mu_{\bar{\lambda}}(x), \mu_{\bar{\gamma}_1}(y_1), \mu_{\bar{\gamma}_2}(y_2), \mu_{\bar{\theta}}(z): A = f(x, y_1, y_2, z)\}$ to satisfying$\mu_{\overline{P_1}}(A) = \alpha, 0 < \alpha \leq 1$.
We consider the following four cases:

Case (i) $\mu_{\bar{\lambda}}(x) = \alpha,\ \mu_{\bar{\mu}}(y) \geq \alpha,\ \mu_{\bar{\theta}}(z) \geq \alpha,\ \mu_{\bar{\beta}}(s) \geq \alpha$

Case (ii) $\mu_{\bar{\lambda}}(x) \geq \alpha,\ \mu_{\bar{\mu}}(y) = \alpha,\ \mu_{\bar{\theta}}(z) \geq \alpha,\ \mu_{\bar{\beta}}(s) \geq \alpha$

Case (iii) $\mu_{\bar{\lambda}}(x) \geq \alpha,\ \mu_{\bar{\mu}}(y) \geq \alpha,\ \mu_{\bar{\theta}}(z) = \alpha,\ \mu_{\bar{\beta}}(s) \geq \alpha$

Case (iv) $\mu_{\bar{\lambda}}(x) \geq \alpha,\ \mu_{\bar{\mu}}(y) \geq \alpha,\ \mu_{\bar{\theta}}(z) \geq \alpha,\ \mu_{\bar{\beta}}(s) = \alpha$.

For case (i) the lower and upper bound of α- cuts of $\overline{P_1}$ can be obtained through the corresponding parametric non-linear programs,

$$[P_1]_\alpha^{L_1} = \min_\Omega\{[A]\} \text{ and } [P_1]_\alpha^{U_1} = \max_\Omega\{[A]\}.$$

Similarly, we can calculate the lower and upper bounds of the α-cuts of $\overline{P_1}$ for the case $(ii), (iii)$and(iv). By considering all the cases simulatuosly the lower and upper bounds of the α-cuts of $\overline{P_1}$ can be written as,

$$A = \left[\frac{\left[\frac{x+z+y_2}{z+y_2} + \frac{z(x+y_2+z)}{x} + \frac{zy_1y_2(zx+z^2+zy_2+x)(z+y_2)^2}{x+z+y_2} + \frac{x^2z(x+z+y_2)y_1}{(xy_2)^2(y_1-x)} + \frac{(x^2+2xz+z^2+zy_2)y_1-x}{x+z+y_2}\right]^{-1}}{\left(\frac{x}{x^2+xy_2xz+x}\right)\left(\frac{y_1-x}{y_1}\right)\left(\frac{z+y_2}{x+z+y_2}\right)}\right]\frac{z}{x}$$

$$[P_1]_\alpha^L = \min_\Omega\{[A]\} \text{ and } [P_1]_\alpha^U = \max_\Omega\{[A]\}.$$

such that$x_\alpha^L \leq x \leq x_\alpha^U, {y_1}_\alpha^L \leq y_1 \leq {y_1}_\alpha^U, {y_2}_\alpha^L \leq y_2 \leq {y_2}_\alpha^U, z_\alpha^L \leq z \leq z_\alpha^U$.
If both $(P_1)_\alpha^L$ and $(P_1)_\alpha^U$ are invertible with respect to α, the left and right shape function, $L(A) = [(P_1)_\alpha^L]^{-1}$ and $R(A) = [(P_1)_\alpha^U]^{-1}$ can be derived from which the membership function $\mu_{\overline{P_1}}(A)$ can be constructed as

$$\mu_{\overline{P_1}}(A) = \begin{cases} L(A), & {P_1}_{\alpha=0}^L \leq A \leq {P_1}_{\alpha=0}^U \\ 1, & {P_1}_{\alpha=1}^L \leq A \leq {P_1}_{\alpha=1}^U \\ R(A), & {P_1}_{\alpha=1}^L \leq A \leq {P_1}_{\alpha=0}^U \end{cases} \tag{7}$$

In the same way we get the following results.

Probability of the server is in a set-up period

$$\mu_{\overline{P_2}}(B) = \begin{cases} L(B), & {P_2}_{\alpha=0}^L \leq B \leq {P_2}_{\alpha=0}^U \\ 1, & {P_2}_{\alpha=1}^L \leq B \leq {P_2}_{\alpha=1}^U \\ R(B), & {P_2}_{\alpha=1}^L \leq B \leq {P_2}_{\alpha=0}^U \end{cases} \tag{8}$$

Probability of the server is in a regular busy period

$$\mu_{\overline{P_3}}(C) = \begin{cases} L(C), \ \ {P_3}^{L}_{\alpha=0} \le C \le {P_3}^{U}_{\alpha=0} \\ 1, {P_3}^{L}_{\alpha=1} \ \ \le C \le {P_3}^{U}_{\alpha=1} \\ R(C), {P_3}^{L}_{\alpha=1} \le C \le {P_3}^{U}_{\alpha=0} \end{cases} \tag{9}$$

Membership function of the mean queue length

$$\mu_{\overline{E(L)}}(D) = \begin{cases} L(D), \ \ (E(L))^{L}_{\alpha=0} \le D \le (E(L))^{U}_{\alpha=0} \\ 1, (E(L))^{L}_{\alpha=1} \le D \le (E(L))^{U}_{\alpha=1} \\ R(D), \ (E(L))^{L}_{\alpha=1} \le D \le (E(L))^{U}_{\alpha=0} \end{cases} \tag{10}$$

Membership function of the mean waiting time

$$\mu_{\overline{E(W)}}(E) = \begin{cases} L(E), \ \ (E(W))^{L}_{\alpha=0} \le E \le (E(W))^{U}_{\alpha=0} \\ 1, (E(W))^{L}_{\alpha=1} \le E \le (E(W))^{U}_{\alpha=1} \\ R(E), \ (E(W))^{L}_{\alpha=1} \le E \le (E(W))^{U}_{\alpha=0} \end{cases} \tag{11}$$

4. Numerical study

Probability of the server is in a close-down period

Suppose the fuzzy arrival rate $\overline{\lambda}$, fuzzy service rate $\overline{\gamma_1}$ in a regular busy period, arriving customers can be served at a mean service rate $\overline{\gamma_2}$, working vacation time $\overline{\theta}$ are assumed to be trapezoidal fuzzy numbers described by: $\overline{\lambda} = [11,12,13,14]$, $\overline{\gamma_1} = [31,32,33,34]$, $\overline{\gamma_2} = [61,62,63,64]$ and $\overline{\theta} = [71,72,73,74]$ permins respectively. Then

$$\lambda(\alpha) = \min_{x \in s(\bar{\lambda})} \{x \in s(\bar{\lambda}), G(x) \ge \alpha\}, \max_{x \in s(\bar{\lambda})} \{x \in s(\bar{\lambda}), \ge \alpha\}, \text{ where,}$$

$G(x) = \begin{cases} x - 11, 11 \le x \le 12 \\ 11, 12 \le x \le 13 \\ 14 - x, 13 \le x \le 14 \end{cases}$, using the above technique we get following results.

Probability of the server is in a close-down period

$$\mu_{\overline{P_1}}(A) = \begin{cases} L(A) \ \ 0.0815 \le A \le 0.2761 \\ 1, 0.2761 \le A \le 0.3918, \\ R(A), 0.3918 \le A \le 0.6583 \end{cases}$$

Probability of the server is in a set-up period

$$\mu_{\overline{P_2}}(B) = \begin{cases} L(B), & 0.1785 \leq B \leq 0.4983 \\ 1,0.4983 \leq B \leq 0.5531, & \\ R(B), & 0.5531 \leq B \leq 0.7581 \end{cases}$$

Probability of the server is in a regular busy period

$$\mu_{\overline{P_3}}(C) = \begin{cases} L(C), & 0.0974 \leq C \leq 0.3671 \\ 1, & 0.3671 \leq C \leq 0.4752, \\ R(C), & 0.4752 \leq C \leq 0.6816 \end{cases}$$

Membership function of the mean queue length

$$\mu_{\overline{E(L)}}(D) = \begin{cases} L(D), & 0.5412 \leq D \leq 1.7153 \\ 1, & 1.7153 \leq D \leq 4.2590. \\ R(D), & 4.2590 \leq D \leq 16.1363 \end{cases}$$

Membership function of the mean waiting time

$$\mu_{\overline{E(W)}}(E) = \begin{cases} L(E), & 0.5761 \leq E \leq 8.6583 \\ 1, & 8.6583 \leq E \leq 12.1107. \\ R(E), & 12.1107 \leq E \leq 19.1265 \end{cases}$$

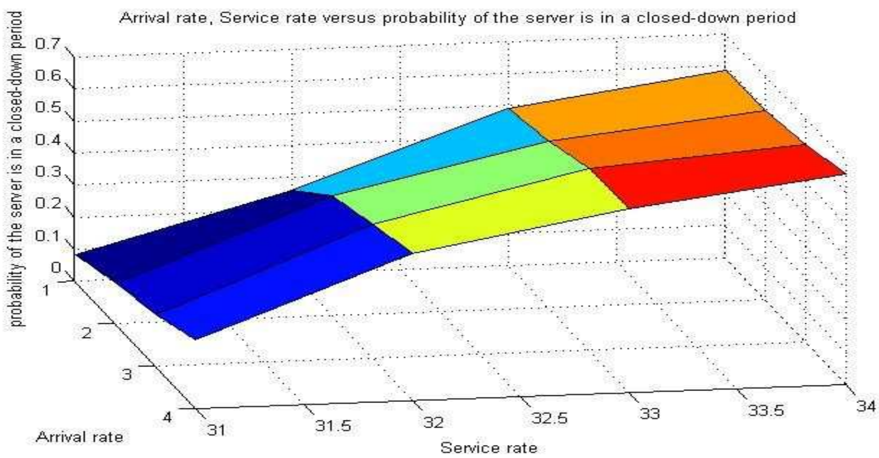

Figure 1: Arrival rate, service rate versus probability of the server is in a closed-down period

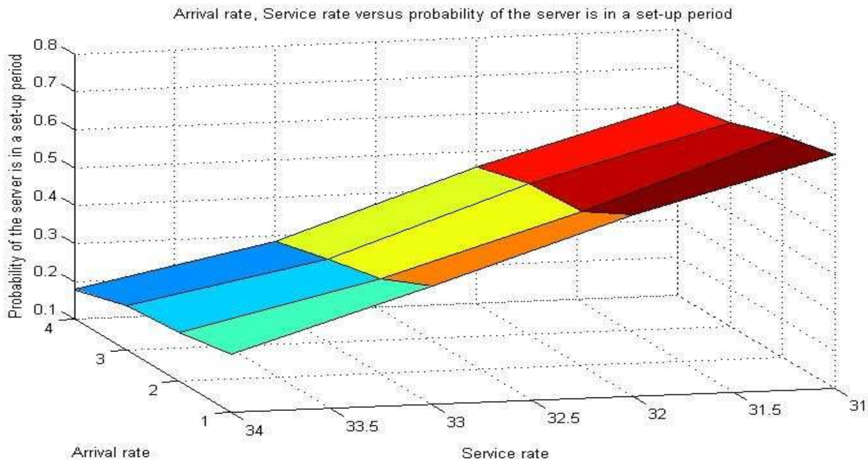

Figure 2: Arrival rate, serevice rate versus probability of the server is in a set-up period

Further by fixed the vacation rate by a crisp value $\bar{\theta} = 76.4$ and $\bar{\gamma}_2 = 61.3$ taking arrival rate $\overline{\lambda} = [11,12,13,14]$, service rate $\overline{\gamma_1} = [31,32,33,34]$ both trapezoidal fuzzy numbers the values of the probability of the server is in a closed-down period are generated and are plotted in the figure 1, it can be observed that as $\overline{\lambda}$ increases the probability of the server is in a closed-down period increases for the fixed value of the service rate, where as for fixed value of arrival rate, probability of the server is in a closed-down period decreases as service rate increases. Similar conclusion can be obtained for the case $\bar{\gamma}_2 = 63.6$, $\bar{\theta} = 73.4$ Again for fixed values of taking$\overline{\lambda} = [11,12,13,14]$, $\overline{\gamma_1} = [31,32,33,34]$ the graphs of probability of the server is in a set-up period are drawn in figure 2 respectively, these figure show that as arrival rate increases that probability of the server is in a set-up period also increases, while the probability of the server is in a set-up period decreases as the service rate increases in both the case.

It is also observed from the data generated that the membership value of the probability of the server is in a closed-down period is 0.57 and the membership value of the probability of the server is in a set-up period 0.63 when the ranges of arrival rate, service rate and the vacation rate lie in the intervals$(12,13.4), (31.5,34.6)$ and $(71.2,73.4)$ respectively.

REFERENCES

1. Y.Baba, Analysis of a GI/M/1 queue with multiple working vacations, *Operations Research Letters*, 33 (2005) 201-209.
2. A.Banik, U.Gupta and S.Pathak, On the GI/M/1/N queue with multiple working vacations-analytic analysis and computation, *Applied Mathematical Modeling*, 31 (2007) 1701-1710.
3. B.T.Doshi, Queueing systems with vacations a survey, *Queuing Systems*: *Theory and Applications*, 1(1) (1986) 29-66.
4. R.Kalyanaraman, N.Thillaigovidan and G.Kannadasan, A single server vacation queue with fuzzy service time and vacation time distributions, *International Journal of Applied Mathematics*, 32(1) (2009) 39–46.

5. R.Kalyanaraman, N.Thillaigovidan and G.Kannadasan, A single server fuzzy queue with group arrivals and server vacation, *Annamalai University Science Journal*, 46 (2010) 99–102.
6. G.Kannadasan and N.Sathiyamoorthi, The FM/FM/1 queue with single working vacation, *International Journal on Recent and Innovation Trends in Computing and Communication*, 5 (2017) 648-654.
7. W.Liu, X.Xu and N.Tian, Some results on the M/M/1 queue with working vacations, *Operations Research Letters*,1(1) (2007) 595-600.
8. J.Li and N.Tian, The M/M/1 queue with working vacations and vacation interruptions, *Journal of Systems Science and Systems Engineering*, 16 (2007) 121-127.
9. L.D.Servi and S.G.Finn, M/M/1 queues with working vacations(M/M/1/WV), *Performance Evaluation*, 50 (2002) 41-52.
10. Shakir Majid and P.Manoharan, Analysis of the M/M/1 queue with single working vacation and vacation interruption, *International Journal of Mathematics Trends and Technology*, 47 (2017) 31-39.
11. C.Sreenivasan, S.R.Chakravarthy and A.Krishnamoorthy, MAP/PH/1 queue with working vacations, vacation interruptions and Policy, *Applied Mathematical Modelling, Applications*, 37(6) (2013) 3879-3893.
12. H.Takagi, Queuing Analysis, *A Foundation of Performance Analysis of Vacation and Priority*, Elsevier Science Publishers, *B.V.*, Amsterdam, The Netherlands, 1 (1991).
13. N.Tian and Z.G.Zhang, *Vacation Queuing Models: Theory and Applications*, Springer-Verlay, New York (2006).
14. M.Zhang and Z.Hou, Performance analysis of M/G/1 queue with working vacation and vacation interruption, *Journal of Computational and Applied Mathematics,* 234 (10) (2006) 2977-2985.