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3−Remainder Cordial Labeling of Cycle Related Graphs

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ABSTRACT

Let G be a (p, q) graph. Let f be a function from V (G) to the set {1, 2, . . . , k} where k is an integer $2 < k \leq |V(G)|$. For each edge uv assign the label r where r is the remainder when f(u) is divided by f(v) (or) f(v) is divided by f(u) according as $f(u) \geq f(v)$ or $f(v) \geq f(u)$. Then the function f is called a k-remainder cordial labeling of G if $|v_f(i) - v_f(j)| \leq 1$, $i, j \in \{1, . . . , k\}$ where $v_f(x)$ denote the number of vertices labeled with x and $|\eta_e - \eta_o| \leq 1$ where η_e and η_o respectively denote the number of edges labeled with an even integers and number of edges labeled with an odd integers. A graph admits a k-remainder cordial labeling is called a k- remainder cordial graph. In this paper we investigate the 3- remainder cordial labeling behavior of the Web graph, Umbrella graph, Dragon graph, Butterfly graph, etc,.

Keywords: Web graph, Umbrella graph, Dragon graph, Butterfly graph

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1. Introduction

Graphs considered here are finite and simple. Graph labeling is used in several areas of science and technology like coding theory, astronomy, circuit design etc. For more details refer Gallian [1]. The origin of graph labeling is graceful labeling which was introduced by Rosa (1967). Ponraj et al. [3,5], introduced remainder cordial labeling of graphs and investigate the remainder cordial labeling behavior of certain graphs, and also the concept of k-remainder cordial labeling introduced [4,6,7] and investigate the 4-remainder cordial labeling behavior of Grid, Subdivision of crown, Subdivision of bistar, Book, Jelly fish, Subdivision of Jelly fish, Mongolian tent, Flower graph, Sunflower graph and Subdivision of Ladder graph, $L_n \odot K_1$, $L_n \odot 2K_1$, $L_n \odot K_2$. Recently, Ponraj et al. [8,9], further introduced the 3-remainder cordial labeling behavior of certain graphs. we prove that path, cycle, star, comb, crown, wheel, fan, square of path, subdivision of wheel, subdivision of star, subdivision of comb, armed crown, $K_{1,n} \odot K_2$ are 3-remainder cordial.

In this paper we investigate the 3-remainder cordial labeling behavior of Web graph, Umbrella graph, Dragon graph, Butterfly graph, etc,. Terms are not defined here follows from Harary [2] and Gallian [1].

2. Preliminary results

Definition 2.1. The butterfly graph ($By_{m,n}$) is a two even cycles of the same order say C_n, sharing a common vertex with m pendant edges attached at the common vertex is called a butterfly graph.

Definition 2.2. The umbrella graph ($U_{n,m}$) is obtained from a fan $F_n = P_n + K_1$ where $P_n = u_1, u_2, \ldots, u_n$ and $V(K_1) = \{u\}$ by pasting the end vertex of the path $P_m = v_1, v_2, \ldots, v_m$ to the vertex of K_1 of the fan F_n.

Definition 2.3. The web graph ($W_{2,n}$) is the graph obtained from a closed Helm CH_n by adding a single pendent edge to each vertex of the outer cycle.

Definition 2.4. A dragon graph is a graph formed by joining an end vertex of a path P_n to a vertex of the cycle C_m. It is denoted as C_m @ P_n.

2. k-remainder cordial labeling

Definition 2.1. Let G be a (p, q) graph. Let f be a function from V (G) to the set {1, 2, . . . , k} where k is an integer $2 < k \leq |V(G)|$. For each edge uv assign the label r where r is the remainder when f(u) is divided by f(v) (or) f(v) is divided by f(u) according as $f(u) \geq f(v)$ or $f(v) \geq f(u)$. The function f is called a k-remainder cordial labeling of G if $|v_f(i) - v_f(j)| \leq 1$, $i, j \in \{1, \ldots, k\}$ where $v_f(x)$ denote the number of vertices labeled with x and $|\eta_e - \eta_o| \leq 1$ where η_e and η_o respectively denote the number of edges labeled with an even integers and number of edges labeled with an odd integers. A graph with a k-remainder cordial labeling is called a k- remainder cordial graph.

First we investigate the 3-remainder cordial labeling behavior of the web graph WG_n.

Theorem 2.2. The web graph WG_n is 3-remainder cordial for all even values of n.

Proof: Let $V(WG_n) = V(C_n \times P_2) \cup \{w_i : 1 \leq i \leq n\}$ and $E(WG_n) = E(C_n \times P_2) \cup \{v_i w_i\}: 1 \leq i \leq n\}$. Then it is easy to verify that WG_n has 3n vertices and 4n edges.

First we consider the vertices u_1, v_1 and w_1. Assign the labels 1 ,2 and 3 respectively to the vertices u_1, v_1 and w_1. Next consider the vertices u_2, v_2 and w_2. Assign the labels 1,3 and 2 to the vertices u_2, v_2 and w_2 respectively. Next we move to the vertices u_3, v_3 and w_3 and assign the labels 1 ,2 and 3 respectively to the vertices u_3, v_3 and w_3. Next assign the labels 1,3 and 2 to the vertices u_4, v_4, w_4. That is assign the labels 1 ,2 ,3 ; 1, 3, 2; …. ; 1, 2 ,3 ; 1,3,2 respectively to the vertices u_1, v_1, w_1 ; u_2, v_2, w_2 ; …. ; $u_{n-1}, v_{n-1}, w_{n-1}$; u_n, v_n, w_n. Note that the vertex condition and edge condition are $v_f(1) = v_f(2) = v_f(3) = n$ and $\eta_e = \eta_o = 2n$ respectively. Hence the function f is 3- remainder cordial labeling behavior of the web graph WG_n all even values of n.

Next we investigate the dragon C_m @ P_n.

Theorem 2.3. The dragon graph C_m @ P_n is 3-remainder cordial for all $m \geq 3$ and n=m.

Proof: Let $C_n= u_1u_2 ….. u_nu_1$ be the cycle and $P_n = v_1 , v_2 ,….. ,v_n$ be the path. Without loss of generality, unify the vertices u_1 with v_1. Clearly the order and size of the dragon C_m @ P_n are 2n-1 and 2n-1 respectively.

Case(1): $n\equiv 0 \pmod 3$ and n is odd.

First consider the vertices $v_1 , v_2 ,….. ,v_n$ of the path. Assign the label 2 to the vertices $v_1 , v_3 ,….. ,v_n$ and 3 to the vertices $v_2 , v_4 ,….. ,v_{n-1}$. Next consider the vertices u_i for $2 \leq i \leq n$. Fix the labels 1, and 1 to the first two vertices u_2, and u_3. Next assign the labels 1,2, and 1 respectively to the next three vertices u_4,u_5, and u_6. Then assign the labels 1,3, and 1 respectively to the next three vertices u_7,u_8, and u_9. Proceeding like this until we reach the vertices u_{n-2},u_{n-1}, and u_n. Clearly the vertices u_{n-2},u_{n-1}, and u_n received the labels 1,3, and 1 for this pattern.

Case(2): $n\equiv 0 \pmod 3$ and n is even.

First consider the vertices v_i of the path. Assign the label 2 to the vertices $v_1 , v_3 ,….. ,v_{n-1}$ and 3 to the vertices $v_2 , v_4 ,….. ,v_n$. Next consider the vertices u_i for $2 \leq i \leq n$. As in case(1), assign the labels to the vertices u_i for $2 \leq i \leq n-3$. Finally assign the labels 1,2, and 1 respectively to the last three vertices u_{n-2},u_{n-1}, and u_n.

Case(3): $n\equiv 1 \pmod 3$ and n is odd.

As in case(1), assign the labels to the vertices v_i of the path for $1 \leq i \leq n$. Next consider the vertices u_i for $2 \leq i \leq n$. Fix the labels 1,3, and 1 to the first three vertices u_2,u_3, and u_4. Next assign the labels 1,3, and 1 respectively to the next three vertices u_5,u_6, and u_7. Then assign the labels 1,2, and 1 respectively to the next three vertices u_8,u_9, and u_{10}. Continuing like this until we reach the vertices u_{n-2},u_{n-1}, and u_n. Then clearly the vertices u_{n-2},u_{n-1}, and u_n received the labels 1,3, and 1 for this pattern.

Case(4): $n\equiv 1 \pmod 3$ and n is even.

As in case(2), assign the labels to the vertices v_i of the path for$1 \leq i \leq n$. Next consider the vertices u_i for $2 \leq i \leq n$. As in case(3), assign the labels to the vertices u_i for $2 \leq i \leq n -3$. Finally assign the labels 1,2, and 1 respectively to the last three vertices u_{n-2},u_{n-1}, and u_n.

Case(5): $n\equiv 2 \pmod 3$ and n is odd.

As in case(1), assign the labels to the vertices v_i of the path for $1 \leq i \leq n$. Next consider the vertices u_i for $2 \leq i \leq n$. Fix the labels 1,1,3, and 1 to the first four vertices u_2,u_3,u_4, and u_5. Next assign the labels 1,2, and 1 respectively to the next three vertices u_6,u_7, and u_8. Then assign the labels 1,3, and 1 respectively to the next three vertices u_9,u_{10}, and u_{11}. Proceeding like this until we reach the vertices u_{n-2},u_{n-1}, and u_n. Then clearly the vertices u_{n-2},u_{n-1}, and u_n received the labels 1,3, and 1 for this pattern.

Case(6): $n\equiv 2 \pmod 3$ and n is even.

As in case(2), assign the labels to the vertices v_i of the path P_n for $1 \leq i \leq n$. Next consider the vertices u_i for $2 \leq i \leq n$. As in case(5), assign the labels to the vertices u_i for $2 \leq i \leq n-3$. Finally assign the labels 1,2, and 1 to the last three vertices u_{n-2}, u_{n-1}, and u_n respectively. The **table -1**, establish that the function f is 3- remainder cordial labeling of the dragon graph C_m @ P_n.

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$	η_e	η_o
n≡ 0 (mod 3)	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{2n-3}{3}$	n	n-1
n≡ 1 (mod 3)	$\frac{2n-2}{3}$	$\frac{2n-2}{3}$	$\frac{2n+1}{3}$	n	n-1
n≡ 2 (mod 3)	$\frac{2n-1}{3}$	$\frac{2n-1}{3}$	$\frac{2n-1}{3}$	n	n-1

Table 1:

Next we investigate the 3-remainder cordial labeling behavior of the butterfly graph $BF_{m,n}$.

Theorem 2.4. The butterfly graph $BF_{m,n}$ is 3-remainder cordial for all $m \geq 3$ and n=m.

Proof: Let $u_1u_2 \ldots.. u_nu_1$ and $v_1 v_2 \ldots.. v_n v_1$ be the two copies of the cycle C_n. Without loss of generality, unify the vertices u_1 and v_1. Let $w_1, w_2, \ldots.., w_n$ be the n pendent vertices. Clearly the order and size of the butterfly graph $BF_{m,n}$ are 3n-1 and 3n respectively.

Case(1): n is odd.

First we consider the vertices u_i of the cycle C_n. Assign the label \$1\$ to the vertices u_i for $2 \leq i \leq n$. Next we consider the vertices v_i of the another cycle C_n. Assign the label 2 to the vertices $v_1, v_3, \ldots.., v_n$. Then next assign the label 3 to the vertices $v_2, v_4, \ldots.., v_{n-1}$. Now consider the pendent vertices w_i for $1 \leq i \leq n$.

Assign the label 3 to the first $\frac{n+1}{2}$ pendent vertices and assign the label 2 to the remaining $\frac{n-1}{2}$ pendent vertices.

Case(2): n is even.

As in case(i), assign the labels to the vertices u_i for $2 \leq i \leq n$. Now we consider the vertices v_i of the another cycle C_n. Assign the label 2 to the vertices $v_1, v_3, \ldots.., v_{n-1}$. Then next assign the label 3 to the vertices $v_2, v_4, \ldots.., v_n$. Now consider the pendent vertices w_i for $1 \leq i \leq n$. Assign the label 3 to the first $\frac{n}{2}$ pendent vertices and assign the label 2 to the remaining $\frac{n}{2}$ pendent vertices. The **table-2**, shows that the function f is 3- remainder cordial labeling of the butterfly graph $BF_{m,n}$.

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$	η_e	η_o
n is odd	n-1	n	N	$\frac{3n+1}{2}$	$\frac{3n-1}{2}$
n is even	n-1	n	N	$\frac{3n}{2}$	$\frac{3n}{2}$

Table 2:

Here we investigate the umbrella graph $U_{m,n}$.

Theorem 2.5. The umbrella graph $U_{m,n}$ is 3-remainder cordial for all $m \geq 3$ and n=m.

Proof: Let $u_1,u_2, \ldots ,u_n$ and $v_1, v_2, \ldots ,v_n$ be the vertices of the umbrella graph $U_{m,n}$. Let $u_1,u_2, \ldots ,u_n$ be the vertices of the path P_n and $v_1, v_2, \ldots ,v_n$ be the vertices of the fan F_n. Without loss of generality, unify the vertices u_1 and v_1. Then clearly the order and size of the umbrella graph $U_{n,n}$ are 2n and 3n-2 respectively.

Case(1): $n \equiv 0 \pmod 3$ and n is even.

Assign the label \$1\$ to the vertices $v_1, v_2, \ldots , v_{n/3}$ and assign the label 3 to the vertices $v_{(n/3)+1}, v_{(n/3)+3}, \ldots, v_{n-1}$ and followed by assign the label 2 to the vertices $v_{(n/3)+2}, v_{(n/3)+4}, \ldots, v_n$. Next consider the vertices u_i of the path P_n. First assign the labels 3,2 and 3 to the vertices u_1,u_2, and u_3 respectively. Next assign the labels 2,1 and 1 to the vertices u_4,u_5, and u_6 respectively. Again assign the labels 3,2 and 3 to the vertices u_7,u_8, and u_9 respectively. Next assign the labels 2,1 and 1 to the vertices u_{10},u_{11}, and u_{12} respectively. Proceeding like this until we reach the vertices u_{n-2},u_{n-1}, and u_n are received the labels 2,1 and 1 respectively.

Case(2): $n \equiv 0 \pmod 3$ and n is odd.

As in case(1), assign the labels to the vertices v_i for $1 \leq i \leq n$, and u_i for $1 \leq i \leq n-3$. Finally assign the labels 3, 2 and 1 to the vertices u_{n-2},u_{n-1}, and u_n respectively.

Case(3): $n \equiv 1 \pmod 3$ and n is odd.

Assign the label 1 to the vertices $v_1, v_2, \ldots , v_{n-1/3}$ and assign the label 3 to the vertices $v_{(n-1/3)+1}, v_{(n-1/3)+3}, \ldots, v_n$, and followed by assign the label 2 to the vertices $v_{(n-1/3)+2}, v_{(n-1/3)+4}, \ldots, v_{n-1}$. Next consider the vertices u_i of the path P_n. Fix the labels 3,2 and 1 to the vertices u_1,u_2, and u_3 and 1,2,3 and 2 to the vertices u_{n-3}, u_{n-2},u_{n-1}, and u_n respectively. First assign the labels 1,2 and 3 to the vertices u_4,u_5, and u_6 respectively. Next assign the labels 2,3 and 1 to the vertices u_7,u_8, and u_9 respectively. Again assign the labels 1,2 and 3 to the vertices u_{10},u_{11}, and u_{12} respectively. Next assign the labels 2,3 and 1 to the vertices u_{13},u_{14}, and u_{15} respectively. Proceeding like this until we reach the vertices u_{n-6},u_{n-5}, and u_{n-4} are received the labels 1,2 and 3 respectively.

Case(4): $n \equiv 1 \pmod 3$ and n is even.

As in case(3), assign the labels to the vertices v_i for $1 \leq i \leq n$. Next assign the labels 2,3 and 1 to the vertices u_1,u_2, and u_3 respectively and as in case(3), assign the labels to the vertices u_i for $4 \leq i \leq n$.

Case(5): $n \equiv 2 \pmod 3$ and n is odd.

Assign the label 1 to the vertices $v_1, v_2, \ldots , v_{n+1/3}$ and assign the label 3 to the vertices $v_{(n+1/3)+1}, v_{(n+1/3)+3}, \ldots, v_n$ and followed by assign the label 2 to the vertices $v_{(n+1/3)+2}, v_{(n+1/3)+4}, \ldots, v_{n-1}$. Next consider the vertices u_i of the path P_n. Fix the labels 3,2 and 1 to the vertices u_1,u_2, and u_3 and 1,2,3 and 2 to the vertices u_{n-3}, u_{n-2},u_{n-1}, and u_n respectively. Assign the labels 2, 3 and 1 to the vertices u_4,u_5, and u_6 respectively. Next assign the labels 1,2 and 3 respectively to the vertices u_7,u_8, and u_9. Again assign the labels 2, 3 and

1 to the vertices u_{10},u_{11}, and u_{12} respectively. Continuing like this until we reach the vertices u_{n-6}, u_{n-5}, and u_{n-4} are received the labels 2,3 and 1 respectively.

Case(6): n≡ 2(mod 3) and n is even.

As in case(5), assign the labels to the vertices v_i for $1 \leq i \leq n$, and assign the labels to the vertices u_i for $1 \leq i \leq n-2$. Finally assign the labels 2 and 3 to the last two vertices u_{n-1} and u_n respectively. The **table -3**, establish that the function f is 3- remainder cordial labeling of the umbrella graph $U_{m,n}$.

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$	η_e	η_o
n≡ 0 (mod 3)& n is odd	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{3n-1}{2}$	$\frac{3n-3}{2}$
n≡ 0 (mod 3)& n is even	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{3n-2}{2}$	$\frac{3n-2}{2}$
n≡ 1 (mod 3)& n is odd	$\frac{2n-2}{3}$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$	$\frac{3n-1}{2}$	$\frac{3n-3}{2}$
n≡ 1 (mod 3)& n is even	$\frac{2n-2}{3}$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$	$\frac{3n-2}{2}$	$\frac{3n-2}{2}$
n≡ 2 (mod 3)& n is odd	$\frac{2n-1}{3}$	$\frac{2n-1}{3}$	$\frac{2n+2}{3}$	$\frac{3n-1}{2}$	$\frac{3n-3}{2}$
n≡ 2 (mod 3)& n is even	$\frac{2n-1}{3}$	$\frac{2n-1}{3}$	$\frac{2n+2}{3}$	$\frac{3n-1}{2}$	$\frac{3n-3}{2}$

Table 3:

3. Conclusion

The main aim of this paper was to present blow-up results for a graph labeling problems subject to certain conditions. The possible generalization is plan to present the sufficient conditions which guarantee the occurrence of the blow-up.

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