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Degree of a Vertex in Complement of Modular Product of Intuitionistic Fuzzy Graphs

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ABSTRACT

In this paper, the complement of modular product of two intuitionistic fuzzy graphs is defined. The degree of a vertex in the complement of modular product of intuitionistic fuzzy graph is studied. Some results on complement of modular product of two regular intuitionistic fuzzy graphs are stated and proved.

Keywords: Modular product, intuitionistic fuzzy graphs, complement.

Mathematical Subject Classification (2010): 05C72

1. Introduction

In 1975, the notion of fuzzy graph and several fuzzy analogues of graph theoretical concepts such as paths, cycles and connectedness are introduced by Rosenfeld [16]. In 2015, Dogra [6] defined some new operations on fuzzy graphs. Nagoor Gani and Fathima Kani [7] introduced the complement of alpha product of fuzzy graphs. The concept of intuitionistic fuzzy set was introduced by Atanassov [3,4] and he added new components that determine the degree of non-membership in the definition of fuzzy set with the condition that the sum of membership and non-membership is less than or equal to one. Intuitionistic fuzzy sets have been applied in a wide variety of areas, including engineering, computer science, medicine, mathematics, chemistry, and economics. In 1994, Shannon and Atanassov [19] introduced the concept of intuitionistic fuzzy graphs. Parvathi and Karunambigai [15] gave a definition for intuitionistic fuzzy graph as a special case of intuitionistic fuzzy graphs defined by Shannon and Atanassov [19]. Akram and Davvaz [1] introduced strong intuitionistic fuzzy graphs. Akram and Al-Shehri [2] studied the intuitionistic fuzzy cycles and intuitionistic fuzzy trees. Yahya and Ali [8,9,10,11] defined some operations on intuitionistic fuzzy graphs and interval-valued Pythagorean fuzzy graphs. Yahya and Ali [12,13] introduced the notion of intuitionistic fuzzy graph metric space and studied some fixed point theorems. Sahoo and Pal [17,18] studied some products of two intuitionistic fuzzy graphs. In this paper, the complement of modular product of two intuitionistic fuzzy graphs and theirs properties are studied. Theorems related to the above concepts are stated and proved.

2. Preliminaries

In this section, we review some basic definitions that are necessary for this paper.

Definition 2.1. [15] An intuitionistic fuzzy graph is of the form $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ on $G^* = (V, E)$, where

1. $V = \{x_1, x_2, \ldots, x_n\}$ such that $\sigma_1 : V \to [0,1]$ and $\sigma_2 : V \to [0,1]$ denote the degree of membership and non-membership of the element $x_i \in V$ respectively, such that $\sigma_1(x_i) + \sigma_2(x_i) \leq 1$ for all $x_i \in V$.
2. $E \subseteq V \times V$ where $\mu_1 : E \to [0,1]$ and $\mu_2 : E \to [0,1]$ are defined by $\mu_1(x_i, x_j) \leq \sigma_1(x_i) \wedge \sigma_1(x_j)$ and $\mu_2(x_i, x_j) \leq \sigma_2(x_i) \vee \sigma_2(x_j)$ such that $\mu_1(x_i, x_j) + \mu_2(x_i, x_j) \leq 1$, $\forall (x_i, x_j) \in E$.

Definition 2.2. [1] An intuitionistic fuzzy graph $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ is called strong intuitionistic fuzzy graph if $\mu_1(x_i, x_j) = \sigma_1(x_i) \wedge \sigma_1(x_j)$ and $\mu_2(x_i, x_j) = \sigma_2(x_i) \vee \sigma_2(x_j), \forall (x_i, x_j) \in E, i \neq j$.

Definition 2.3. [15] An intuitionistic fuzzy graph $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ is called complete intuitionistic fuzzy graph if
$\mu_1(x_i, x_j) = \sigma_1(x_i) \wedge \sigma_1(x_j)$ and$\mu_2(x_i, x_j) = \sigma_2(x_i) \vee \sigma_2(x_j), \forall\, x_i, x_j \in V,\ i \neq j$.

Definition 2.4. [7] Let $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ be an intuitionistic fuzzy graph, then the order of G is defined to be $O(G) = \left(O_{\sigma_1}(G), O_{\sigma_2}(G)\right)$ where $O_{\sigma_1}(G) = \sum_{x \in V} \sigma_1(x)$ and $O_{\sigma_2}(G) = \sum_{x \in V} \sigma_2(x)$.

Definition 2.5. [7] Let $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ be an intuitionistic fuzzy graph, then the size of G is defined to be $S(G) = \left(S_{\mu_1}(G), S_{\mu_2}(G)\right)$ where $S_{\mu_1}(G) = \sum_{xy \in E} \mu_1(xy)$ and $S_{\mu_2}(G) = \sum_{xy \in E} \mu_2(xy)$.

Definition 2.6. [15] The complement of an intuitionistic fuzzy graph $G = (V, E)$ is an intuitionistic fuzzy graph $\bar{G} = (\overline{(\sigma_1, \sigma_2)}, \overline{(\mu_1, \mu_2)})$, where $\overline{(\sigma_1, \sigma_2)} = (\sigma_1, \sigma_2)$ and $\bar{\mu}_1(xy) = \sigma_1(x) \wedge \sigma_2(y) - \mu_1(xy)$ and $\bar{\mu}_2(xy) = \sigma_1(x) \vee \sigma_2(y) - \mu_2(xy)$, $\forall xy \in E$.

Definition 2.7. [7] Let $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ be an intuitionistic fuzzy graph. The degree of a vertex x in G is denoted by $d_G(x) = (d_1^G(x), d_2^G(x))$ and defined by $d_1^G(u) = \sum_{x \neq y} \mu_1^G(x, y) = \sum_{(x,y) \in E} \mu_1^G(x, y)$ and $d_2^G(u) = \sum_{x \neq y} \mu_2^G(x, y) = \sum_{(x,y) \in E} \mu_2^G(x, y)$.

Definition 2.8. [8] Let $G_1 = ((\sigma_1^{G_1}, \sigma_2^{G_1}), (\mu_1^{G_1}, \mu_2^{G_1}))$ and $G_2 = ((\sigma_1^{G_2}, \sigma_2^{G_2}), (\mu_1^{G_2}, \mu_2^{G_2}))$ be two fuzzy graphs with underlying vertex sets V_1 and V_2 and edge sets E_1 and E_2 respectively. Then modular product of G_1 and G_2 is a pair of functions $((\sigma_1^{G_1} \odot \sigma_1^{G_2})(\sigma_2^{G_1} \odot \sigma_2^{G_2}), (\mu_1^{G_1} \odot \mu_1^{G_2})(\mu_2^{G_1} \odot \mu_2^{G_2}))$ with underlying vertex set $V_1 \odot V_2 = \{(x_1, y_1)/x_1 \in V_1, y_1 \in V_2\}$ and underlying edge set $E_1 \odot E_2 = \{(x_1, y_1)(x_2, y_2)/\ x_1x_2 \in E_1, y_1y_2 \in E_2 \quad or\ x_1x_2 \notin E_1, y_1, y_2 \notin E_2\}$ with

$\left(\sigma_1^{G_1} \odot \sigma_1^{G_2}\right)(x_1, y_1) = \sigma_1^{G_1}(x_1) \wedge \sigma_1^{G_2}(y_1),$

$\left(\sigma_2^{G_1} \odot \sigma_2^{G_2}\right)(x_1, y_1) = \sigma_2^{G_1}(x_1) \vee \sigma_2^{G_2}(y_1), where\ x_1 \in V_1\ and\ y_1 \in V_2.$

$$\left(\mu_1^{G_1} \odot \mu_1^{G_2}\right)\left((x_1, y_1), (x_2, y_2)\right) = \begin{cases} \mu_1^{G_1}(x_1x_2) \wedge \mu_1^{G_2}(y_1y_2), & if\ x_1x_2 \in E_1, y_1y_2 \in E_2 \\ \sigma_1^{G_1}(x_1) \wedge \sigma_1^{G_1}(x_2) \wedge \sigma_1^{G_2}(y_1) \wedge \sigma_1^{G_2}(y_2) & if\ \ x_1\ x_2 \notin E_1, y_1y_2 \notin E_2. \end{cases}$$

$$(\mu_2^{G_1} \odot \mu_2^{G_2})((x_1, y_1), (x_2, y_2)) = \begin{cases} \mu_2^{G_1}(x_1x_2) \vee \mu_2^{G_2}(y_1y_2), & if\ x_1x_2 \in E_1, y_1y_2 \in E_2 \\ \sigma_2^{G_1}(x_1) \vee \sigma_2^{G_1}(x_2) \vee \sigma_2^{G_2}\ (y_1) \vee \sigma_2^{G_2}(y_2) & if\ \ x_1x_2 \notin E_1, y_1y_2 \notin E_2 \end{cases}$$

Theorem 2.1. [8] Let $G_1 = ((\sigma_1^{G_1}, \sigma_2^{G_1}), (\mu_1^{G_1}, \mu_2^{G_1}))$ and $G_2 = (\left(\sigma_1^{G_2}, \sigma_2^{G_2}\right), \left(\mu_1^{G_2}, \mu_2^{G_2}\right))$ be two complete intuitionistic fuzzy graphs.

1. If $\mu_1^{G_1} \leq \mu_1^{G_2}, \mu_2^{G_1} \geq \mu_2^{G_2}$, then $d^{G_1 \odot G_2}(x, y) = d^{G_1}(x)$, where $d^{G_1 \odot G_2}(x, y) = \left(d_1^{G_1 \odot\ G_2}(x, y), d_2^{G_1 \odot G_2}(x, y)\right)$ and $d^{G_1}(x) = (d_1^{G_1}(x), d_2^{G_1}(x))$
2. If $\mu_1^{G_1} \geq \mu_1^{G_2}, \mu_2^{G_1} \leq \mu_2^{G_2}, then\ d^{G_1 \odot\ G_2}\ (x, y) = d^{G_2}(x),$ $where\ d^{G_1 \odot G_2}(x, y) = \left(d_1^{G_1 \odot G_2}(x, y), d_2^{G_1 \odot G_2}(x, y)\right)$ and $$d^{G_2}(x) = (d_1^{G_2}(x), d_2^{G_2}(x))$$

Theorem 2.2. [8] Let $G_1 = \left(\left(\sigma_1^{G_1}, \sigma_2^{G_1}\right), \left(\mu_1^{G_1}, \mu_2^{G_1}\right)\right)$ and $G_2 = \left(\left(\sigma_1^{G_2}, \sigma_2^{G_2}\right), \left(\mu_1^{G_2}, \mu_2^{G_2}\right)\right)$ be two intuitionistic fuzzy graphs with underlying crisp graphs G_1^* is complete and G_2^* is regular. If $\sigma_1^{G_1} \geq \mu_1^{G_2}$, $\sigma_2^{G_1} \leq \mu_2^{G_2}$; $\sigma_1^{G_2} \geq \mu_1^{G_1}$, $\sigma_2^{G_2} \leq \mu_2^{G_1}$ and $\mu^{G_1} = \mu^{G_2}$, then $G_1 \odot G_2$ is a regular if and only if G_1 is a regular intuitionistic fuzzy graph.

3. Complement of modular product of intuitionistic fuzzy graphs

Definition 3.1. The complement of modular product of two intuitionistic fuzzy graphs $G_1 = ((\sigma_1^{G_1}, \sigma_2^{G_1}), (\mu_1^{G_1}, \mu_2^{G_1}))$ and $G_2 = ((\sigma_1^{G_2}, \sigma_2^{G_2}), (\mu_1^{G_2}, \mu_2^{G_2}))$ is an intuitionistic fuzzy graphs

$\overline{G_1 \odot G_2} = \left(\left(\overline{\sigma_1^{G_1} \odot \sigma_1^{G_2}}\right)\left(\overline{\sigma_2^{G_1} \odot \sigma_2^{G_2}}\right), \left(\overline{\mu_1^{G_1} \odot \mu_1^{G_2}}\right)\left(\overline{\mu_2^{G_1} \odot \mu_2^{G_2}}\right)\right)$ on $G^* = (V, E)$and $\overline{V_1 \odot V_2} = V_1 \odot V_2$and

$$\overline{E_1 \circledcirc E_2} = \left\{ (x_1, y_1)(x_2, y_2) \middle| \begin{array}{c} x_1 = x_2, y_1y_2 \in E_2\ (or) y_1 = y_2, x_1x_2 \in E_1\ (or) \\ x_1x_2 \in E_1, y_1y_2 \notin E_2\ (or)\ x_1x_2 \notin E_1, y_1y_2 \in E_2\ (or) \\ x_1x_2 \in E_1, y_1y_2 \in E_2\ (or)\ x_1x_2 \notin E_1, y_1y_2 \notin E_2 \end{array} \right\}$$

$$\left(\overline{\sigma_1^{G_1} \circledcirc \sigma_1^{G_2}}\right)(x_1, y_1) = \left(\sigma_1^{G_1} \circledcirc \sigma_1^{G_2}\right)(x_1, y_1) = \sigma_1^{G_1}(x_1) \wedge \sigma_1^{G_2}(y_1),$$

$$\left(\overline{\sigma_2^{G_1} \circledcirc \sigma_2^{G_2}}\right)(x_1, y_1) = \left(\sigma_2^{G_1} \circledcirc \sigma_2^{G_2}\right)(x_1, y_1) = \sigma_2^{G_1}(x_1) \vee \sigma_2^{G_2}(y_1),$$

where $x_1 \in V_1\ and\ y_1 \in V_2$

$$\left(\overline{\mu_1^{G_1} \circledcirc \mu_1^{G_2}}\right)\left((x_1, y_1), (x_2, y_2)\right) =$$

$$\begin{cases} \left(\sigma_1^{G_1} \circledcirc \sigma_1^{G_2}\right)(x_1, y_1) \wedge \left(\sigma_1^{G_1} \circledcirc \sigma_1^{G_2}\right)(x_2, y_2) & if\ x_1 = x_2, y_1y_2 \in E_2 \\ \left(\sigma_1^{G_1} \circledcirc \sigma_1^{G_2}\right)(x_1, y_1) \wedge \left(\sigma_1^{G_1} \circledcirc \sigma_1^{G_2}\right)(x_2, y_2) & if\ y_1 = y_2, x_1x_2 \in E_1 \\ \left(\sigma_1^{G_1} \circledcirc \sigma_1^{G_2}\right)(x_1, y_1) \wedge \left(\sigma_1^{G_1} \circledcirc \sigma_1^{G_2}\right)(x_2, y_2) & if\ x_1x_2 \in E_1, y_1y_2 \notin E_2 \\ \left(\sigma_1^{G_1} \circledcirc \sigma_1^{G_2}\right)(x_1, y_1) \wedge \left(\sigma_1^{G_1} \circledcirc \sigma_1^{G_2}\right)(x_2, y_2) & if\ x_1x_2 \notin E_1, y_1y_2 \in E_2 \\ \left(\sigma_1^{G_1} \circledcirc \sigma_1^{G_2}\right)(x_1, y_1) \wedge \left(\sigma_1^{G_1} \circledcirc \sigma_1^{G_2}\right)(x_2, y_2) - \left(\mu_1^{G_1} \circledcirc \mu_1^{G_2}\right)\left((x_1, y_1), (x_2, y_2)\right) & if\ x_1x_2 \in E_1, y_1y_2 \in E_2 \\ \left(\sigma_1^{G_1} \circledcirc \sigma_1^{G_2}\right)(x_1, y_1) \wedge \left(\sigma_1^{G_1} \circledcirc \sigma_1^{G_2}\right)(x_2, y_2) - \left(\mu_1^{G_1} \circledcirc \mu_1^{G_2}\right)\left((x_1, y_1), (x_2, y_2)\right) & if\ x_1x_2 \notin E_1, y_1y_2 \notin E_2 \end{cases}$$

$$\left(\overline{\mu_2^{G_1} \circledcirc \mu_2^{G_2}}\right)\left((x_1, y_1), (x_2, y_2)\right) =$$

$$\begin{cases} \left(\sigma_2^{G_1} \circledcirc \sigma_2^{G_2}\right)(x_1, y_1) \vee \left(\sigma_2^{G_1} \circledcirc \sigma_2^{G_2}\right)(x_2, y_2) & if\ x_1 = x_2, y_1y_2 \in E_2 \\ \left(\sigma_2^{G_1} \circledcirc \sigma_2^{G_2}\right)(x_1, y_1) \vee \left(\sigma_2^{G_1} \circledcirc \sigma_2^{G_2}\right)(x_2, y_2) & if\ y_1 = y_2, x_1x_2 \in E_1 \\ \left(\sigma_2^{G_1} \circledcirc \sigma_2^{G_2}\right)(x_1, y_1) \vee \left(\sigma_2^{G_1} \circledcirc \sigma_2^{G_2}\right)(x_2, y_2) & if\ x_1x_2 \in E_1, y_1y_2 \notin E_2 \\ \left(\sigma_2^{G_1} \circledcirc \sigma_2^{G_2}\right)(x_1, y_1) \vee \left(\sigma_2^{G_1} \circledcirc \sigma_2^{G_2}\right)(x_2, y_2) & if\ x_1x_2 \notin E_1, y_1y_2 \in E_2 \\ \left(\sigma_2^{G_1} \circledcirc \sigma_2^{G_2}\right)(x_1, y_1) \vee \left(\sigma_2^{G_1} \circledcirc \sigma_2^{G_2}\right)(x_2, y_2) - \left(\mu_2^{G_1} \circledcirc \mu_2^{G_2}\right)\left((x_1, y_1), (x_2, y_2)\right) & if\ x_1x_2 \in E_1, y_1y_2 \in E_2 \\ \left(\sigma_2^{G_1} \circledcirc \sigma_2^{G_2}\right)(x_1, y_1) \vee \left(\sigma_2^{G_1} \circledcirc \sigma_2^{G_2}\right)(x_2, y_2) - \left(\mu_2^{G_1} \circledcirc \mu_2^{G_2}\right)\left((x_1, y_1), (x_2, y_2)\right) & if\ x_1x_2 \notin E_1, y_1y_2 \notin E_2 \end{cases}$$

Theorem 3.1
Let G_1 and G_2 be two regular intuitionistic fuzzy graphs. If underlying crisp graphs G_1^* and G_2^* are complete graphs and $\sigma_1^{G_1}, \sigma_2^{G_1}, \sigma_1^{G_2}, \sigma_2^{G_2}$ are constants which satisfies $\sigma_1^{G_1} \geq \mu_1^{G_2}$, $\sigma_2^{G_1} \leq \mu_2^{G_2}$; $\sigma_1^{G_2} \geq \mu_1^{G_1}$, $\sigma_2^{G_2} \leq \mu_2^{G_1}$; $\sigma_1^{G_1} > \mu_1^{G_1}$, $\sigma_2^{G_1} < \mu_2^{G_1}$ and $\sigma_1^{G_2} > \mu_1^{G_2}$, $\sigma_2^{G_2} < \mu_2^{G_2}$. Then complement of modular product of two intuitionistic fuzzy graphs G_1 and G_2 is regular intuitionistic fuzzy graph.

Proof:
Let G_1 and G_2 be two regular intuitionistic fuzzy graphs. The underlying crisp graphs G_1^* and G_2^* are complete graphs of degrees (d_1, d_2) and (d_3, d_4) for every vertices of V_1 and V_2.
Given that $\sigma_1^{G_1}, \sigma_2^{G_1}, \sigma_1^{G_2}$ and $\sigma_2^{G_2}$ are constants, say $\sigma_1^{G_1}(x) = c_1$, $\sigma_2^{G_1}(x) = c_2$, $\forall\, x \in V_1$ $\sigma_1^{G_2}(y) = c_3$, $\sigma_2^{G_2}(y) = c_4$ $\forall\, y \in V_2$ and $\sigma_1^{G_1} \geq \mu_1^{G_2}$, $\sigma_2^{G_1} \leq \mu_2^{G_2}$; $\sigma_1^{G_2} \geq \mu_1^{G_1}$, $\sigma_2^{G_2} \leq \mu_2^{G_1}$.
By theorem, modular product of two regular intuitionistic fuzzy graphs is regular intuitionistic fuzzy graph.
Consider $(x_1, y_1) \in \left(\overline{\sigma_1^{G_1} \odot \sigma_1^{G_2}}\right)$

$$d_1^{\overline{G_1 \odot G_2}}(x_1, y_1) = \sum_{(x_1,y_1)(x_2,y_2)\in E} \left(\overline{\mu_1^{G_1} \odot \mu_1^{G_2}}\right)\big((x_1, y_1)(x_2, y_2)\big)$$

$$= \sum_{x_1=x_2, y_1y_2\in E_1} \left(\sigma_1^{G_1} \odot \sigma_1^{G_2}\right)(x_1, y_1) \wedge \left(\sigma_1^{G_1} \odot \sigma_1^{G_2}\right)(x_2, y_2)$$
$$+ \sum_{y_1=y_2, x_1x_2\in E_1} \left(\sigma_1^{G_1} \odot \sigma_1^{G_2}\right)(x_1, y_1) \wedge \left(\sigma_1^{G_1} \odot \sigma_1^{G_2}\right)(x_2, y_2)$$
$$+ \sum_{x_1x_2\in E_1, y_1y_2\notin E_2} \left(\sigma_1^{G_1} \odot \sigma_1^{G_2}\right)(x_1, y_1) \wedge \left(\sigma_1^{G_1} \odot \sigma_1^{G_2}\right)(x_2, y_2)$$
$$+ \sum_{x_1x_2\notin E_1, y_1y_2\in E_2} \left(\sigma_1^{G_1} \odot \sigma_1^{G_2}\right)(x_1, y_1) \wedge \left(\sigma_1^{G_1} \odot \sigma_1^{G_2}\right)(x_2, y_2)$$
$$+ \sum_{x_1x_2\in E_1, y_1y_2\in E_2} \left(\sigma_1^{G_1} \odot \sigma_1^{G_2}\right)(x_1, y_1) \wedge \left(\sigma_1^{G_1} \odot \sigma_1^{G_2}\right)(x_2, y_2)$$
$$- \left(\mu_1^{G_1} \odot \mu_1^{G_2}\right)\big((x_1, y_1)(x_2, y_2)\big)$$
$$+ \sum_{x_1x_2\notin E_1, y_1y_2\notin E_2} \left(\sigma_1^{G_1} \odot \sigma_1^{G_2}\right)(x_1, y_1) \wedge \left(\sigma_1^{G_1} \odot \sigma_1^{G_2}\right)(x_2, y_2)$$
$$- \left(\mu_1^{G_1} \odot \mu_1^{G_2}\right)\big((x_1, y_1)(x_2, y_2)\big)$$

Since G_1^* and G_2^* are complete graphs. Then

$$d_1^{\overline{G_1 \odot G_2}}(x_1, y_1) = \sum_{x_1=x_2, y_1y_2\in E_2} \left(\sigma_1^{G_1} \odot \sigma_1^{G_2}\right)(x_1, y_1) \wedge \left(\sigma_1^{G_1} \odot \sigma_1^{G_2}\right)(x_2, y_2)$$

$$+\sum_{y_1=y_2,x_1x_2\in E_1}\left(\sigma_1^{G_1}\circledcirc\sigma_1^{G_2}\right)(x_1,y_1)\wedge\left(\sigma_1^{G_1}\circledcirc\sigma_1^{G_2}\right)(x_2,y_2)$$

$$+\sum_{x_1x_2\in E_1,y_1y_2\in E_2}\left(\sigma_1^{G_1}\circledcirc\sigma_1^{G_2}\right)(x_1,y_1)\wedge\left(\sigma_1^{G_1}\circledcirc\sigma_1^{G_2}\right)(x_2,y_2)-\left(\mu_1^{G_1}\circledcirc\mu_1^{G_2}\right)\left((x_1,y_1)(x_2,y_2)\right)\qquad(3.1)$$

Similarly,

$$d_2^{\overline{G_1\circledcirc G_2}}(x_1,y_1)=\sum_{x_1=x_2,y_1y_2\in E_2}\left(\sigma_2^{G_1}\circledcirc\sigma_2^{G_2}\right)(x_1,y_1)\vee\left(\sigma_2^{G_1}\circledcirc\sigma_2^{G_2}\right)(x_2,y_2)$$

$$+\sum_{y_1=y_2,x_1x_2\in E_1}\left(\sigma_2^{G_1}\circledcirc\sigma_2^{G_2}\right)(x_1,y_1)\vee\left(\sigma_2^{G_1}\circledcirc\sigma_2^{G_2}\right)(x_2,y_2)$$

$$+\sum_{x_1x_2\in E_1,y_1y_2\in E_2}\left(\sigma_2^{G_1}\circledcirc\sigma_2^{G_2}\right)(x_1,y_1)\vee\left(\sigma_2^{G_1}\circledcirc\sigma_2^{G_2}\right)(x_2,y_2)-\left(\mu_2^{G_1}\circledcirc\mu_2^{G_2}\right)\left((x_1,y_1)(x_2,y_2)\right)\qquad(3.2)$$

Case $(\boldsymbol{i})$: If $\sigma_1^{G_1}(x)\le\sigma_1^{G_2}(y)$ and $\sigma_2^{G_1}(x)\ge\sigma_2^{G_2}(y)$; $\mu_1^{G_1}\le\mu_1^{G_2}$ $\mu_2^{G_1}\ge\mu_2^{G_2}$ for all $x\in V_1,\ y\in V_2$

By Equation 3.1,

$$d_1^{\overline{G_1\circledcirc G_2}}(x_1,y_1)=\sum_{x_1=x_2,y_1y_2\in E_2}\left(\sigma_1^{G_1}(x_1)\wedge\sigma_1^{G_2}(y_1)\right)\wedge\left(\sigma_1^{G_1}(x_2)\wedge\sigma_1^{G_2}(y_2)\right)$$

$$+\sum_{y_1=y_2,x_1x_2\in E_1}\left(\sigma_1^{G_1}(x_1)\wedge\sigma_1^{G_2}(y_1)\right)\wedge\left(\sigma_1^{G_1}(x_2)\wedge\sigma_1^{G_2}(y_2)\right)$$

$$\sum_{x_1x_2\in E_1,y_1y_2\in E_2}\left(\sigma_1^{G_1}(x_1)\wedge\sigma_1^{G_2}(y_1)\right)\wedge\left(\sigma_1^{G_1}(x_2)\wedge\sigma_1^{G_2}(y_2)\right)-\left(\mu_1^{G_1}\circledcirc\mu_1^{G_2}\right)\left((x_1,y_1)(x_2,y_2)\right)$$

By Equation 3.2,

$$d_2^{\overline{G_1\circledcirc G_2}}(x_1,y_1)=\sum_{x_1=x_2,y_1y_2\in E_2}\left(\sigma_2^{G_1}(x_1)\vee\sigma_2^{G_2}(y_1)\right)\vee\left(\sigma_2^{G_1}(x_2)\vee\sigma_2^{G_2}(y_2)\right)$$

$$+ \sum_{y_1=y_2, x_1x_2 \in E_1} \left(\sigma_2^{G_1}(x_1) \vee \sigma_2^{G_2}(y_1)\right) \vee \left(\sigma_2^{G_1}(x_2) \vee \sigma_2^{G_2}(y_2)\right)$$

$$+ \sum_{x_1x_2 \in E_1, y_1y_2 \in E_2} \left(\sigma_2^{G_1}(x_1) \vee \sigma_2^{G_2}(y_1)\right) \vee \left(\sigma_2^{G_1}(x_2) \vee \sigma_2^{G_2}(y_2)\right) - \left(\mu_2^{G_1} \circledcirc \mu_2^{G_2}\right)\left((x_1, y_1)(x_2, y_2)\right)$$

Since by the definition of modular product of two intuitionistic fuzzy graphs,

$$d_1^{\overline{G_1 \circledcirc G_2}}(x_1, y_1) = \sum_{x_1=x_2, y_1y_2 \in E_2} \sigma_1^{G_1}(x_1) + \sum_{y_1=y_2, x_1x_2 \in E_1} \sigma_1^{G_1}(x_1) + \sum_{x_1x_2 \in E_1, y_1y_2 \in E_2} c_1 - \left(\mu_1^{G_1} \circledcirc \mu_1^{G_2}\right)\left((x_1, y_1)(x_2, y_2)\right)$$

$$= \sum_{x_1=x_2, y_1y_2 \in E_2} c_1 + \sum_{y_1=y_2, x_1x_2 \in E_1} c_1 + \left(c_1 - \mu_1^{G_1}(x_1, x_2)\right) d_{G_2}^*(y_1) d_{G_1}^*(x_1)$$

$$d_1^{\overline{G_1 \circledcirc G_2}}(x_1, y_1) = c_1 d_{G_2}^*(y_1) + c_1 d_{G_1}^*(x_1) + (c_1 - e_1) d_{G_2}^*(y_1) d_{G_1}^*(x_1)$$

Similarly,

$$d_2^{\overline{G_1 \circledcirc G_2}}(x_1, y_1) = c_2 d_{G_2}^*(y_1) + c_2 d_{G_1}^*(x_1) + (c_2 - e_2) d_{G_2}^*(y_1) d_{G_1}^*(x_1)$$

Since G_1 and G_2 are two regular intuitionistic fuzzy graphs & G_1^* and G_2^* are complete graphs, then $\mu_1^{G_1}$ and $\mu_1^{G_2}$ are constants say (e_1, e_2) and (e_3, e_4).

$$d_1^{\overline{G_1 \circledcirc G_2}}(x_1, y_1) = c_1 d_2 + c_1 d_1 + (c_1 - e_1) d_1 d_2,$$

$$d_2^{\overline{G_1 \circledcirc G_2}}(x_1, y_1) = c_2 d_2 + c_2 d_1 + (c_2 - e_2) d_1 d_2.$$

Case $(\boldsymbol{ii})$: If $\sigma_1^{G_1}(x) \geq \sigma_1^{G_2}(y)$ and $\sigma_2^{G_1}(x) \leq \sigma_2^{G_2}(y)$; $\mu_1^{G_1} \geq \mu_1^{G_2}$ $\mu_2^{G_1} \leq \mu_2^{G_2}$ for all $x \in V_1$ and $y \in V_2$

$$d_1^{\overline{G_1 \circledcirc G_2}}(x_1, y_1) = \sum_{x_1=x_2, y_1y_2 \in E_1} \sigma_1^{G_2}(x_1) + \sum_{y_1=y_2, x_1x_2 \in E_2} \sigma_1^{G_2}(x_1) + \sum_{x_1x_2 \in E_1, y_1y_2 \in E_2} \sigma_1^{G_2}(x_1) - \left\{\mu_1^{G_1}(x_1, x_2) \wedge \mu_1^{G_2}(y_1, y_2)\right\}$$

$$= \sum_{x_1=x_2, y_1y_2 \in E_1} c_3 + \sum_{y_1=y_2, x_1x_2 \in E_2} c_3 + \left(c_1 - \mu_1^{G_2}(y_1, y_2)\right) d_{G_2}^*(y_1) d_{G_1}^*(x_1)$$

$$= c_3 d_2 + c_3 d_1 + (c_3 - e_3) d_1 d_2$$

Similarly,

$$d_2^{\overline{G_1 \circledcirc G_2}}(x_1, y_1) = c_4 d_2 + c_4 d_1 + (c_4 - e_4) d_1 d_2$$

Hence, complement of modular product of two regular intuitionistic fuzzy graphs is regular.

Theorem 3.2. Let $G_1$and G_2 be two regular intuitionistic fuzzy graphs. If underlying crisp graphs G_1^* and G_2^* are regular graphs and σ^{G_1}, $\sigma^{G_2}, \mu^{G_1}$and μ^{G_2} are different constants which satisfies $\sigma_1^{G_1} > \mu_1^{G_2}$, $\sigma_2^{G_1} < \mu_2^{G_2}$; $\sigma_1^{G_2} > \mu_1^{G_1}$, $\sigma_2^{G_2} < \mu_2^{G_1}$; $\sigma_1^{G_1} > \mu_1^{G_1}$, $\sigma_2^{G_1} < \mu_2^{G_1}$ and $\sigma_1^{G_2} > \mu_1^{G_2}$, $\sigma_2^{G_2} < \mu_2^{G_2}$. Then complement of the modular product of two regular intuitionistic fuzzy graphs G_1 and G_2 is regular intuitionistic fuzzy graph.

Proof: Let G_1 and G_2 be two regular intuitionistic fuzzy graphs. The underlying crisp graphs G_1^* and G_2^* are regular graphs of degrees (d_1, d_2) and (d_3, d_4) for every vertices in V_1 and V_2.

Given that $\sigma^{G_1}, \sigma^{G_2}$, μ^{G_1} and μ^{G_2} are constants, say $\sigma_1^{G_1}(x) = c_1$, $\sigma_2^{G_1}(x) = c_2$, $\forall\, x \in V_1$ $\sigma_1^{G_2}(y) = c_3$, $\sigma_2^{G_2}(y) = c_4$ $\forall\, y \in V_2$, $\mu_1^{G_1}(x_1y_1) = e_1, \mu_2^{G_1}(x_1y_1) = e_2$, $\mu_1^{G_2}(x_1y_1) = e_3, \mu_2^{G_2}(x_1y_1) = e_4$ and $\sigma_1^{G_1} > \mu_1^{G_2}$, $\sigma_2^{G_1} < \mu_2^{G_2}$; $\sigma_1^{G_2} > \mu_1^{G_1}$, $\sigma_2^{G_2} < \mu_2^{G_1}$.

Consider $(x_1, y_1) \in \left(\overline{\sigma_1^{G_1} \odot \sigma_1^{G_2}}\right)$

Case $(\boldsymbol{i})$: If $\sigma_1^{G_1}(x) \le \sigma_1^{G_2}(y)$ and $\sigma_2^{G_1}(x) \ge \sigma_2^{G_2}(y)$; $\mu_1^{G_1} \le \mu_1^{G_2}$, $\mu_2^{G_1} \ge \mu_2^{G_2}$ for all $x \in V_1$ and $y \in V_2$

$d_1^{\overline{G_1 \odot G_2}}(x_1, y_1) = \sum_{x_1 = x_2, y_1y_2 \in E_2} \left(\sigma_1^{G_1}(x_1) \wedge \sigma_1^{G_2}(y_1)\right) \wedge \left(\sigma_1^{G_1}(x_2) \wedge \sigma_1^{G_2}(y_2)\right)$
$+ \sum_{y_1 = y_2, x_1x_2 \in E_1} \left(\sigma_1^{G_1}(x_1) \wedge \sigma_1^{G_2}(y_1)\right) \wedge \left(\sigma_1^{G_1}(x_2) \wedge \sigma_1^{G_2}(y_2)\right)$
$+ \sum_{x_1x_2 \in E_1, y_1y_2 \notin E_2} \left(\sigma_1^{G_1}(x_1) \wedge \sigma_1^{G_2}(y_1)\right) \wedge \left(\sigma_1^{G_1}(x_2) \wedge \sigma_1^{G_2}(y_2)\right) +$
$\sum_{x_1x_2 \notin E_1, y_1y_2 \in E_2} \left(\sigma_1^{G_1}(x_1) \wedge \sigma_1^{G_2}(y_1)\right) \wedge \left(\sigma_1^{G_1}(x_2) \wedge \sigma_1^{G_2}(y_2)\right) +$
$\sum_{x_1x_2 \notin E_1, y_1y_2 \notin E_2} \left(\sigma_1^{G_1}(x_1) \wedge \sigma_1^{G_2}(y_1)\right) \wedge \left(\sigma_1^{G_1}(x_2) \wedge \sigma_1^{G_2}(y_2)\right) - \big(\mu_1^{G_1} \odot$
$\mu_1^{G_2}\big)\big((x_1, y_1)(x_2, y_2)\big) + \sum_{x_1x_2 \in E_1, y_1y_2 \in E_2} \left(\sigma_1^{G_1}(x_1) \wedge \sigma_1^{G_2}(y_1)\right) \wedge \Big(\sigma_1^{G_1}(x_2) \wedge$
$\sigma_1^{G_2}(y_2)\Big)$ $-\big(\mu_1^{G_1} \odot \mu_1^{G_2}\big)\big((x_1, y_1)(x_2, y_2)\big)$

$= \sum_{x_1 = x_2, y_1y_2 \in E_2} \sigma_1^{G_1}(x_1) + \sum_{x_1 = x_2, y_1y_2 \in E_2} \sigma_1^{G_1}(x_1) + \sum_{x_1x_2 \in E_1, y_1y_2 \notin E_2} \sigma_1^{G_1}(x_1) +$
$\sum_{x_1x_2 \in E_1, y_1y_2 \notin E_2} \sigma_1^{G_1}(x_1) + \sum_{x_1x_2 \notin E_1, y_1y_2 \notin E_2} \big(\sigma_1^{G_1}(x_1) - \{\sigma_1^{G_1}(x_1) \wedge \sigma_1^{G_1}(x_2) \wedge$
$\sigma_1^{G_2}(y_1) \wedge \sigma_1^{G_2}(y_2)\}\big) + \sum_{x_1x_2 \in E_1, y_1y_2 \in E_2} \left(\sigma_1^{G_1}(x_1) - \mu_1^{G_1}(x_1x_2) \wedge \mu_1^{G_2}(y_1y_2)\right)$

$= \sum_{x_1 = x_2, y_1y_2 \in E_2} \sigma_1^{G_1}(x_1) + \sum_{x_1 = x_2, y_1y_2 \in E_2} \sigma_1^{G_1}(x_1) + \sum_{x_1x_2 \in E_1, y_1y_2 \notin E_2} \sigma_1^{G_1}(x_1) +$
$\sum_{x_1x_2 \notin E_1, y_1y_2 \in E_2} \sigma_1^{G_1}(x_1) + (c_1 - e_1)d_1d_2$

$$d_1^{\overline{G_1 \odot G_2}}(x_1, y_1) = c_1r_2 + c_1r_1 + c_1|\overline{E_2}| + c_1d_2|\overline{E_1}| + (c_1 - e_1)d_1d_2.$$

Similarly,

$$d_2^{\overline{G_1 \odot G_2}}(x_1, y_1) = c_2r_2 + c_2r_1 + c_2|\overline{E_2}| + c_2d_2|\overline{E_1}| + (c_2 - e_2)d_1d_2$$

This is true for all vertices of $\overline{V_1 \odot V_2}$.

Case (ii): If $\sigma_1^{G_2}(x) \le \sigma_1^{G_1}(y)$ and $\sigma_2^{G_2}(x) \ge \sigma_2^{G_1}(y)$; $\mu_1^{G_1} \ge \mu_1^{G_2}$, $\mu_2^{G_1} \le \mu_2^{G_2}$ for all $x \in V_1$ and $y \in V_2$.

$$d_1^{\overline{G_1 \odot G_2}}(x_1, y_1) = \sum_{x_1=x_2, y_1y_2 \in E_2} \sigma_1^{G_2}(x_2) + \sum_{x_1=x_2, y_1y_2 \in E_2} \sigma_1^{G_2}(x_2) + \sum_{x_1x_2 \in E_1, y_1y_2 \notin E_2} \sigma_1^{G_2}(x_2) + \sum_{x_1x_2 \notin E_1, y_1y_2 \in E_2} \sigma_1^{G_2}(x_1) + \sum_{x_1x_2 \notin E_1, y_1y_2 \notin E_2} \left(\sigma_1^{G_2}(x_1) - \left\{\sigma_1^{G_1}(x_1) \wedge \sigma_1^{G_1}(x_2) \wedge \sigma_1^{G_2}(y_1) \wedge \sigma_1^{G_2}(y_2)\right\}\right) + \sum_{x_1x_2 \in E_1, y_1y_2 \in E_2} \left(\sigma_1^{G_2}(x_1) - \mu_1^{G_1}(x_1x_2) \wedge \mu_1^{G_2}(y_1y_2)\right)$$

where $|\overline{E_1}|$ and $|\overline{E_2}|$ are the degree of the vertex of complement graphs G_1^* and G_2^*.

$$d_1^{\overline{G_1 \odot G_2}}(x_1, y_1) = c_3 d_2 + c_3 d_1 + c_3 d_1 |\overline{E_2}| + c_3 d_1 |\overline{E_1}| + (c_3 - e_3) d_1 d_2$$

Similarly,

$$d_2^{\overline{G_1 \odot G_2}}(x_1, y_1) = c_4 d_2 + c_4 d_1 + c_4 d_1 |\overline{E_2}| + c_4 d_1 |\overline{E_1}| + (c_4 - e_4) d_1 d_2$$

This is true for all vertices in $\overline{V_1 \odot V_2}$.

Hence complement of modular product of two regular intuitionistic fuzzy graphs is regular.

4. Conclusion

Intuitionistic fuzzy sets are very useful in providing a flexible model to describe uncertainty and vagueness involved in decision making, so intuitionistic fuzzy graphs are playing a substantial role in chemistry, economics, computer sciences, engineering, medicine and decision making problems, now a days. In this paper, the complement of modular product is defined. The degree of a vertex in complement of modular product of intuitionistic fuzzy graph is studied. Some results on complement of regular modular product of intuitionistic fuzzy graphs are stated and proved.

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