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On Fuzzy F' - Spaces

G. Thangaraj and S. Muruganantham

Thiruvalluvar University Serkkadu, 632115, India.
E-mail: g.thangaraj@rediffmail.com

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ABSTRACT

In this paper, the concept of fuzzy F'-spaces is introduced and studied. Several characterizations of fuzzy F'-Spaces are established. It is established that fuzzy perfectly disconnected spaces are fuzzy F'-Spaces. The conditions for a fuzzy basically disconnected space to become a fuzzy extremally disconnected space is obtained. A condition for a fuzzy F'-space to become a fuzzy basically disconnected space is also obtained in this paper.

Keywords: fuzzy dense set, fuzzy G_δ -set , fuzzy F_σ -set, fuzzy σ–boundary set, fuzzy pseudo open set, fuzzy regular open set, fuzzy perfectly disconnected space, fuzzy extremally disconnected space, fuzzy basically disconnected space.

Mathematical Subject Classification (2010): 54A40, 03E72

1. Introduction

The notion of fuzzy sets as a new approach to a mathematical representation of vagueness in everyday language, was introduced by Zadeh [13] in classical paper in the year 1965. The potential of fuzzy notion was realized by the researcher and has successfully been applied in all branches of mathematics. In 1968, Chan [4] introduced the concept of fuzzy topological spaces. The paper of Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. In the recent years, there has been a growing trend among topologies spaces to introduce and study different forms of fuzzy spaces in fuzzy topology. In classical topology,F'-spaces were introduced by Leonard Gillman and Henrilsen[5], in which disjoint cozero-sets have disjoint closures. Motivated on these lines, the concept of the fuzzy F'- space is introduced in this paper. Several characterizations of fuzzy F'- spaces are established. It is established that fuzzy perfectly disconnected spaces are fuzzy F'-spaces. The conditions for a fuzzy basically disconnected space to become a fuzzy extremally disconnected space is obtained. A condition for a fuzzy F'-space to become a fuzzy basically disconnected space is also obtained in this paper.

2. Preliminaries

In order to make the exposition self-contained, some basic notions and results used in the sequel, are given. In this work by (X,T) or simply by X, we will denote a fuzzy topological space due to Chang (1968). Let X be a non-empty set and I the unit interval [0,1]. A fuzzy set λ in X is a mapping from X into I. The fuzzy set 0_X is defined as $0_X (x) = 0$, for all $x \in X$ and the fuzzy set 1_X is defined as $1_X (x) = 1$, for all $x \in X$. The complement of a fuzzy set λ on X is given by $\lambda^C = 1 - \lambda$.

Definition 2.1. [4] Let (X,T) be a fuzzy topological space and λ be any fuzzy set in (X,T). The interior and the closure of λ , are defined respectively as follows :

(i). int $(\lambda) = \vee \{ \mu / \mu \leq \lambda , \mu \in T\}$

(ii). cl$(\lambda) = \wedge \{ \mu / \lambda \leq \mu , 1-\mu \in T \}$.

Definition 2.2. A fuzzy set λ in a fuzzy topological space (X,T), is called a

(i). ***fuzzy dense set*** if there exists no fuzzy closed set μ in (X,T) such that $\lambda < \mu < 1$. That is, cl $(\lambda) = 1$, in (X,T) **[7]** .

(ii). ***fuzzy*** $\boldsymbol{G_\delta}$ ***- set*** in (X, T) if $\lambda = \wedge_{i=1}^{\infty} (\lambda_i)$, where $\lambda_i \in T$ for $i \in I$ **[3].**

(iii). ***fuzzy*** $\boldsymbol{F_\sigma}$ ***- set*** in (X, T) if $\lambda = \vee_{i=1}^{\infty} (\lambda_i)$, where $1 - \lambda_i \in T$ for $i \in I$ **[3]**.

(iv). ***fuzzy regular-open*** if $\lambda =$ int cl(λ) and fuzzy regular-closed if $\lambda =$ cl int(λ)**[1].**

(v). ***fuzzy σ-nowhere dense set*** if λ is a fuzzy F_σ-set in (X, T) such that Int $(\lambda) = 0$ [**12**].

(vi). ***fuzzy σ-boundary set*** if $\lambda = \vee_{i=1}^{\infty} (\mu_i)$, where $\mu_i =$ cl $(\lambda_i) \wedge (1-\lambda_i)$ and (λ_i)'s are fuzzy regular open sets in (X, T) [**11**] .

(vii). ***fuzzy*** $\boldsymbol{Pseudo-open}$ ***set*** if $\lambda = \mu \vee \delta$, where μ is a non-zero fuzzy open set in (X, T) and δ is a fuzzy first category set in (X, T) [**7**] .

Lemma 2.1. [1] For a fuzzy set λ of a fuzzy topological space (X, T). Then $1 - \text{int}(\lambda) = \text{cl}(1-\lambda)$ and $1 - \text{cl}(\lambda) = \text{int}(1-\lambda)$.

Definition 2.3. [7] A fuzzy set λ in a topological space (X,T) is called a fuzzy first category set if $\lambda = \vee_{k=1}^{\infty} (\lambda_k)$, where $(\lambda_k)'s$ are fuzzy nowhere dense sets in (X,T). Any other fuzzy set in (X,T) is said to be of fuzzy second category.

Definition 2.4. A fuzzy topological space (X,T) is called a

(i). ***fuzzy submaximal space*** if for each fuzzy set λ in (X, T) such that cl $(\lambda) = 1$, $\lambda \in T$ in (X, T) [**3**].

(ii)**.** ***fuzzy perfectly disconnected space*** if for any two non–zero fuzzy sets λ and μ

defined on X with $\lambda \leq 1 - \mu$, then $cl(\lambda) \leq 1 - cl(\mu)$, in (X, T) [**9**].

(iii). ***fuzzy basically disconnected space*** if the closure of each fuzzy open F_σ - set is a fuzzy open in (X, T) [**6**].

(iv). ***fuzzy globall disconnected space*** if each fuzzy semi-open set in (X,T) is a fuzzy open [**10**].

(v). ***fuzzy extremally disconnected space*** if $\lambda \in T$ implies that $cl(\lambda) \in T$[**2**].

Theorem 2.1. [8] If λ is a fuzzy Pseudo open set in a fuzzy submaximal space (X, T), then λ = μ ∨ δ, where μ is a fuzzy open set and δ is a fuzzy F_σ - set in (X, T).

Theorem 2.2. [11] If λ is a fuzzy σ-boundary set in a fuzzy topological space (X, T), then λ is a fuzzy F_σ- set in (X, T).

Theorem 2.3. [10] If λ is a fuzzy first category set in a fuzzy globally disconnected space (X, T), then λ is a fuzzy F_σ- set in (X, T).

Theorem 2.4. [6] For any fuzzy topological space (X,T), the following are equivalent;

(i) . X is fuzzy basically disconnected.

(ii) . for each fuzzy closed G_δ-set λ, int(λ) is fuzzy closed.

(iii). for each fuzzy open F_σ-set λ, cl(λ) + cl[1- cl(λ)] = 1.

Theorem 2.5. [1] In a fuzzy topological space

(a). The closure of a fuzzy open set is a fuzzy regular closed set, and

(b). the interior of a fuzzy closed set is a fuzzy regular open set.

3. Fuzzy $\mathbf{F}'$ −Spaces

Definition 3.1. A topological space (X, T) is called a fuzzy F'- space if $\lambda \leq 1 - \mu$, where λ and μ are fuzzy F_σ– sets in (X, T), then $cl(\lambda) \leq 1 - cl(\mu)$, in (X, T).

Example 3.1. Let X = { a, b, c}. Consider the fuzzy sets α, β, γ, μ and δ defined on X as follows:

α : X →[0, 1] is defined as α(a) = 0.5 ; α(b) = 0.6 ; α(c) = 0.4,

β : X →[0, 1] is defined as β(a) = 0.4 ; β(b) = 0.5 ; β(c) = 0.6,

γ : X →[0, 1] is defined as γ(a) = 0.6 ; γ(b) = 0.4 ; γ(c) = 0.5.

Then, T = { 0, α, β, γ, α ∨ β, α ∨ γ , β ∨ γ, α ∧ β, α ∧ γ , β ∧ γ , α ∨ (β ∨ γ), α ∨ (β ∧ γ), α ∧ (β ∧ γ), α ∧ (β ∨ γ), β ∨ (α ∧ γ), β ∧ (α ∨ γ), γ ∨ (α ∧ β), γ ∧ (α ∨ β), 1} is a fuzzy topology on X. On computation, λ = {1 - [α ∨ β]} ∨ {1-[α ∨ (β ∧ γ)]} ∨ {1 – [β ∨ (α ∧ γ)]} and μ = {1- (α ∨ γ)} ∨ {1 – (β ∨ γ)} ∨{1 - α ∨ (β ∨

$\gamma)\} \vee \{1 - \gamma \vee (\alpha \wedge \beta)\}$, and then λ and μ are fuzzy F_σ-sets in (X, T). Also on computation cl (λ) = 1 – [β ∧ (α ∨ γ)], cl (μ) = 1 – [γ ∨ (α ∧ β)] in (X, T). Now, $\lambda \leq 1 - \mu$ implies that $cl(\lambda) \leq 1 - cl(\mu)$ in (X, T) implies that (X, T) is a fuzzy F'-space.

Proposition 3.1. If $\lambda + \mu \leq 1$, for any two fuzzy F_σ-sets λ and μ in a fuzzy F'-space (X, T), then $cl(\lambda) + cl(\mu) \leq 1$, in (X, T).

Proof: Suppose that $\lambda + \mu \leq 1$, where λ and μ are fuzzy F_σ-sets in (X, T). Then $\lambda \leq 1 - \mu$, in (X, T). Since (X, T) is a fuzzy F'-space, $cl(\lambda) \leq 1 - cl(\mu)$ in (X, T) and then $cl(\lambda) + cl(\mu) \leq 1$, in (X, T).

Proposition 3.2. If δ and η are any two fuzzy G_δ-sets such that $1 - \eta \leq \delta$ in a fuzzy F'- space, then

i). $1 - int(\eta) \leq int(\delta)$ in (X, T).

ii). $1 - \eta \leq int(\delta)$, in (X, T).

Proof: i). Let δ and η are any two fuzzy G_δ-sets in (X, T) such that $1 - \eta \leq \delta$. Now $1 - \delta \leq 1 - (1 - \eta)$ in (X, T), where $(1 - \delta)$ and $(1 - \eta)$ are fuzzy F_σ- sets in (X, T). Since (X, T) is a fuzzy F'-space, $cl(1 - \delta) \leq 1 - cl(1 - \eta)$ in (X, T). Then, $1 - int(\delta) \leq 1 - [1 - int(\eta)]$ and then $1 - int(\delta) \leq int(\eta)$, in (X, T). Hence $1 - int(\eta) \leq int(\delta)$, in (X, T).

ii). By (i), $1 - int(\eta) \leq int(\delta)$ in (X, T). Now $1 - \eta \leq 1 - int(\eta)$ implies that $1 - \eta \leq int(\delta)$, in (X, T).

Proposition 3.3. If (X, T) is a fuzzy F'-space and $\lambda \leq 1 - \mu$ for any two non-zero fuzzy F_σ- sets λ and μ in (X, T), then λ and μ are not fuzzy dense sets in (X,T).

Proof: Suppose that $\lambda \leq 1 - \mu$ for any two non-zero fuzzy F_σ-sets λ and μ defined on X. Since (X,T) is a fuzzy F'-space, $cl(\lambda) \leq 1 - cl(\mu)$ in (X, T). If $cl(\lambda) = 1$ then $1 \leq 1 - cl(\mu)$ implies that $cl(\mu) \leq 0$. That is, $cl(\mu) = 0$ in (X, T). This will imply that $\mu = 0$ a contradiction to $\mu \neq 0$. If $cl(\mu) = 1$ then $cl(\lambda) \leq 1 - 1 = 0$. That is, $cl(\lambda) = 0$ in (X, T). This will imply that $\lambda = 0$, a contradiction to $\lambda \neq 0$. Thus, $cl(\lambda) \neq 1$ and $cl(\mu) \neq 1$ in (X, T). Hence λ and μ are not fuzzy dense sets in (X, T).

Proposition 3.4. If $\lambda \leq 1 - \mu$ for any two fuzzy F_σ-sets in a fuzzy F'- space (X, T), then

(i). $\lambda + cl(\mu) \leq 1$, in (X, T).

(ii). $\mu + cl(\lambda) \leq 1$, in (X, T).

Proof: Suppose that $\lambda \leq 1 - \mu$, where λ and μ are fuzzy F_σ-sets in (X, T). Since (X, T) is a fuzzy F'-space, $cl(\lambda) \leq 1 - cl(\mu)$ in (X, T). Now $1 - cl(\mu) \leq 1 - \mu$ and $\lambda \leq cl(\lambda)$ implies that $\lambda \leq cl(\lambda) \leq 1 - cl(\mu) \leq 1 - \mu$ in (X, T). Then

(i). $\lambda + cl(\mu) \leq 1$, in (X, T).

(ii). $\mu + cl(\lambda) \leq 1$, in (X, T).

Proposition 3.5. If (X, T) is a fuzzy F'- space in which each fuzzy open set is a fuzzy F_σ- set, then for a fuzzy set λ defined on X, $cl\, int(\lambda) \leq int\, cl(\lambda)$.

Proof: Let λ be a fuzzy set in (X, T). Then $int(\lambda) \leq cl(\lambda)$ implies that $int(\lambda) \leq 1 - [1 - cl(\lambda)]$ in (X, T). Since $int(\lambda)$ and $1 - cl(\lambda)$ are fuzzy open sets in (X, T), by hypothesis, $int(\lambda)$ and $1 - cl(\lambda)$ are fuzzy F_σ- sets in (X, T). Since (X, T) is a fuzzy F'-space $cl\, int(\lambda) \leq 1 - cl[1 - cl(\lambda)]$ in (X,T). Then, $clint(\lambda) \leq 1 - [1 - intcl(\lambda)]$ and thus $cl\, int(\lambda) \leq int\, cl(\lambda)$.

Proposition 3.6. If (X, T) is a fuzzy F'- space and $\lambda \leq 1 - \mu$, where λ and μ are fuzzy σ – nowhere dense sets in (X, T), then $cl(\lambda) \leq 1 - cl(\mu)$, where $int(\lambda) = 0$; $cl(\lambda) \neq 1$ and $int(\mu) = 0$; $cl(\mu) \neq 1$, in (X, T).

Proof: Suppose that $\lambda \leq 1 - \mu$, for any two fuzzy σ–nowhere dense sets λ and μ in (X, T). Then λ and μ are fuzzy F_σ–sets in (X, T) such that $int(\lambda) = 0$and $int(\mu) = 0$in (X, T). Since (X, T) is a fuzzy F'-space, $cl(\lambda) \leq 1 - cl(\mu)$ in (X, T). By proposition 3.3, λ and μ are not fuzzy dense sets in (X, T). Thus, $cl(\lambda) \leq 1 - cl(\mu)$, where $int(\lambda) = 0$; $cl(\lambda) \neq 1$ and $int(\mu) = 0$; $cl(\mu) \neq 1$, in (X, T).

Proposition 3.7. If λ is a fuzzy pseudo - open set in a fuzzy submaximal (X, T), then there exists fuzzy F_σ-set η in (X, T) such that $\lambda \geq \eta$, in (X, T).

Proof: Let λ is a fuzzy pseudo–open set in (X, T). Then, by theorem 2.1, $\lambda = \mu \vee \eta$, where $\mu \in T$ and η is a fuzzy F_σ-set in (X, T). This implies that $\lambda \geq \eta$, in (X, T).

Proposition 3.8. If $\lambda \leq 1 - \mu$, for any two fuzzy pseudo-open sets in a fuzzy submaximal and fuzzy F'-space (X, T), then there exists fuzzy F_σ- sets α and β in (X, T) such that

i). $cl(\alpha) \leq 1 - int(\beta)$, in (X, T).

ii). $\alpha \leq cl(\lambda) \leq 1 - int(\mu) \leq 1 - \beta$, in (X, T).

Proof: Suppose that $\lambda \leq 1 - \mu$, where λ and μ are fuzzy pseudo open sets in (X, T).

Since (X, T) is a fuzzy submaximal space, for the fuzzy pseudo open sets λ and μ, there exist fuzzy F_σ-sets α and β in (X, T), such that $\alpha \leq \lambda$ and $\beta \leq \mu$. Then, $\alpha \leq \lambda \leq 1 - \mu \leq 1 - \beta$ and thus $\alpha \leq 1 - \beta$ in (X, T). Since (X, T) is a fuzzy F'-space, $cl(\alpha) \leq 1 - cl(\beta)$ in (X, T). Now $\alpha \leq \lambda$ implies that $cl(\alpha) \leq cl(\lambda)$, in (X, T) and then $\alpha \leq cl(\lambda)$. Also $\lambda \leq 1 - \mu$ implies that $cl(\lambda) \leq cl(1 - \mu)$ and then $cl(\lambda) \leq 1 - int(\mu)$. Now $\beta \leq \mu$ implies that $int(\beta) \leq int(\mu)$ and then $1 - int(\beta) \geq 1 - int(\mu)$. Thus $\alpha \leq cl(\lambda) \leq 1 - int(\mu) \leq 1 - int(\beta) \leq 1 - \beta$, in (X, T). Thus, we have

i). $cl(\alpha) \leq 1 - int(\beta)$, in (X, T).

ii). $\alpha \leq cl(\lambda) \leq 1 - int(\mu) \leq 1 - \beta$, in (X, T).

Proposition 3.9. If $\lambda \geq \mu$, where $\lambda(\neq 0)$ is a fuzzy G_δ– set and μ is a fuzzy F_σ–set in a fuzzy F'- space (X, T), then there exists fuzzy open set δ in (X, T) such that $cl(\mu) \leq \delta \leq cl(\lambda)$, in (X, T).

Proof: Suppose $\lambda \geq \mu$, where $\lambda(\neq 0)$ is a fuzzy G_δ – set and μ is a fuzzy F_σ–set in (X, T). Then $(1 - \lambda) \leq 1 - \mu$ and $1 - \lambda$ and μ are fuzzy F_σ-sets in (X, T). Since (X, T) is a fuzzy F'-space, $cl(1 - \lambda) \leq 1 - cl(\mu)$ in (X, T) and this implies that $int(\lambda) \geq cl(\mu)$ in (X, T). But $int(\lambda) \leq cl(\lambda)$ implies that $cl(\mu) \leq int(\lambda) \leq cl(\lambda)$, in (X, T). Let $\delta = int(\lambda)$. Then δ is a fuzzy open set in (X, T) such that $cl(\mu) \leq \delta \leq cl(\lambda)$ in (X, T).

Proposition 3.10. If $\lambda \leq \mu$, where λ is a fuzzy F_σ-set and $\mu(\neq 0)$ is a fuzzy G_δ–set in a fuzzy F'-space, then there exists fuzzy closed set η in (X, T) such that $int(\lambda) \leq \eta \leq int(\mu)$, in (X, T).

Proof: Suppose $\lambda \leq \mu$, where λ is a fuzzy F_σ-set and μ is a fuzzy G_δ-set in (X, T). Now, $\lambda \leq 1 - (1 - \mu)$ and λ and $1 - \mu$ are fuzzy F_σ-sets in (X, T). Since (X, T) is a fuzzy F'- space, $cl(\lambda) \leq 1 - cl(1 - \mu)$ and hence $cl(\lambda) \leq int(\mu)$, in (X, T). But $int(\lambda) \leq cl(\lambda)$ implies that $int(\lambda) \leq cl(\lambda) \leq int(\mu)$ in (X, T). Let $\eta = cl(\lambda)$. Thus η is a fuzzy closed set in (X, T) such that, $int(\lambda) \leq \eta \leq int(\mu)$, in (X, T).

Proposition 3.11. If $\lambda \leq 1 - \mu$, where λ and μ are fuzzy σ-boundary sets in a fuzzy F'-space, then $cl(\lambda) \leq 1 - cl(\mu)$ in (X, T).

Proof: Let λ and μ be any two fuzzy σ-boundary sets in (X, T) such that $\lambda \leq 1 - \mu$. Now, by theorem 2.2, λ and μ are fuzzy F_σ-sets in (X, T). Since (X, T) is a fuzzy F'-space, $\lambda \leq 1 - \mu$, in (X, T) implies that $cl(\lambda) \leq 1 - cl(\mu)$ in (X, T).

Proposition 3.12. If $\lambda \leq 1 - \mu$, where λ and μ are fuzzy first category sets in a fuzzy globally disconnected and fuzzy F'-space (X, T), then $cl(\lambda) \leq 1 - cl(\mu)$, in (X, T).
Proof: Let λ and μ are fuzzy first category sets in a fuzzy globally disconnected space (X, T) then by theorem 2.3, λ and μ are fuzzy F_σ-sets in (X, T). Also since (X, T) is a fuzzy F'-space, $\lambda \leq 1 - \mu$ in (X, T) implies that $cl(\lambda) \leq 1 - cl(\mu)$, in (X, T).

Proposition 3.13. If (X, T) is a fuzzy F'-space in which each fuzzy open set is a fuzzy F_σ-set, then for a fuzzy set λ defined on X, there exists a fuzzy regular open set δ in (X, T) such that clint(λ) ≤ δ.
Proof: Let (X, T) be a fuzzy F'-space. Then, by proposition 3.5, for a fuzzy set λ defined on X, clint(λ) ≤ intcl(λ), in (X, T). Since the interior of a fuzzy closed set is a fuzzy regular open set in a fuzzy topological space, theorem 2.5, int[cl(λ)] is a fuzzy regular open set in (X, T). Let δ = intcl(λ). Thus, there exists a fuzzy regular open set δ in (X, T) such that clint(λ) ≤ δ.

Proposition 3.14. If (X, T) is a fuzzy F'-space in which fuzzy open sets are fuzzy F_σ-sets, then there exists a fuzzy regular closed set η and a fuzzy regular open set δ in (X, T) such that η ≤ δ.
Proof: Let (X, T) be a fuzzy F'-space in which fuzzy open sets are fuzzy F_σ-sets. Then, by proposition 3.5, for a fuzzy set λ defined on X, clint(λ) ≤ intcl(λ). Let η = clint(λ) and δ = intcl(λ). Then, by theorem 2.5, η is a fuzzy regular closed set and δ is a fuzzy regular open set in (X, T). Thus, there exists a fuzzy regular closed set η and a fuzzy regular open set δ in (X, T) such that η ≤ δ.

Proposition 3.15. If (X, T) is a fuzzy F'-space in which fuzzy open sets are fuzzy F_σ-sets, then there exists a fuzzy F_σ-set η and fuzzy open set δ in (X, T) such that η ≤ δ.
Proof: Let $(\lambda_i)'s$ be fuzzy sets defined on X. Since (X, T) is a fuzzy F'-space in which fuzzy open sets are fuzzy F_σ- sets, by proposition 3.5, clint(λ_i) ≤ intcl(λ_i). Let η_i = clint(λ_i) and δ_i = intcl(λ_i). Then η_i is a fuzzy regular closed set and hence η_i is a fuzzy closed set in (X, T); δ_i is a fuzzy regular open set and hence δ_i is a fuzzy open set in (X, T). Now $\eta_i \leq \delta_i$, implies that $\vee_{i=1}^{\infty}(\eta_i) \leq \vee_{i=1}^{\infty}(\delta_i)$. Since $(\eta_i)'s$ are fuzzy closed sets, $\vee_{i=1}^{\infty}(\eta_i)$ is a fuzzy F_σ-set in (X, T). Also $\vee_{i=1}^{\infty}(\delta_i)$ is a fuzzy open set in (X, T). Let η = $\vee_{i=1}^{\infty}(\eta_i)$ and δ = $\vee_{i=1}^{\infty}(\delta_i)$. Thus, η ≤ δ, where η is a fuzzy F_σ-set and δ is a fuzzy open set in (X, T).

Proposition 3.16. If (X, T) is a fuzzy F'-space in which fuzzy open sets are fuzzy F_σ-sets, then there exists a fuzzy open and fuzzy F_σ-set δ and fuzzy F_σ-set η in (X, T) such that η ≤ δ.

Proof: Let (X,T) be a fuzzy F'-space in which fuzzy open sets are fuzzy F_σ-sets. Then by proposition 3.15, then there exists a fuzzy F_σ-set η and a fuzzy open set δ in (X, T) such that η ≤ δ. Since δ is a fuzzy open set, by hypothesis, δ is a fuzzy F_σ-set in (X, T). Thus, there exist a fuzzy F_σ- set η and a fuzzy open and fuzzy F_σ- set δ in (X, T) such that η ≤ δ.

4. Fuzzy $\mathbf{F}'$ −spaces and other topological spaces

Proposition 4.1. If a fuzzy topological space (X, T) is called a fuzzy perfectly disconnected space, then (X, T) is a fuzzy F'- space.

Proof: Let (X, T) be a fuzzy perfectly disconnected space and suppose that $\lambda \leq 1 - \mu$, where λ and μ are fuzzy F_σ-sets in (X, T). Since (X, T) is a fuzzy perfectly disconnected space, $\lambda \leq 1 - \mu$, implies that $cl(\lambda) \leq 1 - \mathrm{cl}(\mu)$, in (X, T) and hence (X, T) is a fuzzy F'- space.

Proposition 4.2. If (X, T) is a fuzzy basically disconnected space in which fuzzy open sets are fuzzy F_σ- sets, then for a fuzzy open set λ defined on X, clint(λ) ≤ intcl(λ) in (X, T).

Proof: Let λ be a fuzzy set in (X, T). Now 1− cl(λ) ≤ 1− int(λ), in (X, T). By hypothesis 1 − cl(λ) and int(λ) are fuzzy F_σ-sets in (X, T). Since (X, T) is a fuzzy perfectly disconnected space, by proposition 4.1, (X, T) is a fuzzy F'- space. Then, cl[1 − cl(λ)] ≤ 1 − cl[int(λ)], in (X, T). This implies that 1 − intcl(λ) ≤ 1 − clint(λ) and thus clint(λ) ≤ intcl(λ) in (X, T).

Proposition 4.3. If (X, T) is a fuzzy basically disconnected space in which fuzzy open sets are fuzzy F_σ-set, then (X, T) is a fuzzy extremally disconnected space.

Proof: Let (X, T) be a fuzzy basically disconnected space and λ be a fuzzy open set in (X, T). By hypothesis, λ is a fuzzy F_σ–set in (X, T). Then by theorem 2.4, $cl(\lambda)$ + cl[1 − cl(λ)] = 1, in (X, T). This implie that $cl(\lambda) + 1 - \mathrm{intcl}(\lambda) = 1$ and then $cl(\lambda) = \mathrm{intcl}(\lambda)$ in (X, T). This implies that $cl(\lambda)$ is a fuzzy open set in (X, T) and hence (X, T) is a fuzzy extremally disconnected space.

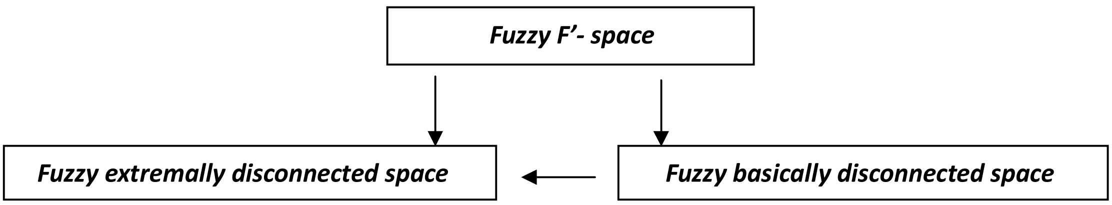
Proposition 4.4. If (X, T) is a fuzzy F'-space in which fuzzy open sets are fuzzy F_σ-sets, then (X, T) is a fuzzy basically disconnected space.

Proof: Let λ be a fuzzy open set in (X, T). By hypothesis, λ is a fuzzy F_σ-set in (X, T) and thus λ is a fuzzy open and fuzzy F_σ-set in (X, T). Since (X, T) is a fuzzy F'-space in which fuzzy open sets are fuzzy F_σ-sets, by proposition 4.3, (X, T) is a fuzzy extremally disconnected space. Then, for the fuzzy open set λ in (X, T) and cl(λ) is a fuzzy open set in (X, T). Thus, for the fuzzy open and fuzzy F_σ–set λ, cl(λ) ϵ T implies that (X, T) is a fuzzy basically disconnected space.

Proposition 4. 5. If (X, T) is a fuzzy perfectly disconnected space in which fuzzy open sets are fuzzy F_σ-sets, then (X, T) is a fuzzy basically disconnected space.
Proof: Let (X, T) is a fuzzy perfectly disconnected space. Then, by proposition 4.1, (X, T) is a fuzzy F'-space. Thus, (X, T) is a fuzzy F'-space in which fuzzy open sets are fuzzy F_σ-sets. Then by proposition 4.4, (X, T) is a fuzzy basically disconnected space.

Proposition 4.6. If (X, T) is a fuzzy F'-space in which fuzzy open sets are fuzzy F_σ-sets, then (X, T) is a fuzzy extremally disconnected space.
Proof: Let λ be a fuzzy open set in the fuzzy F'-space in which fuzzy open sets are fuzzy F_σ-set in (X, T). Then proposition 3.5, $clint(\lambda) \leq intcl(\lambda)$ in (X, T). Since λ is a fuzzy open set, $int(\lambda) = \lambda$ in (X, T) and thus $cl(\lambda) \leq intcl(\lambda)$. But $intcl(\lambda) \leq cl(\lambda)$. Hence $intcl(\lambda) = cl(\lambda)$ and then $cl(\lambda)$ is a fuzzy open set in (X, T) and hence (X, T) is a extremally disconnected space.

Remark 4.1. If fuzzy open sets in fuzzy F_σ -sets is a fuzzy topological space , then one will have the following implications;



REFERENCES

1. K.K.Azad, On fuzzy semi-continuity fuzzy almost continuity and fuzzy weakly continuity, J. Math. Anal. Appl., 82 (1981) 14 - 32.
2. G.Balasubramanian, Fuzzy disconnectedness and its fuzzy stronger forms, Indian J. Pure Appl. Math., 24(1) (1993) 27 - 30.
3. G.Balasubramanian, Maximal fuzzy topologies, Kybernetika, 5(31) (1995) 459-464.
4. C.L.Chang, Fuzzy topological spaces, J. Math. Anal. Appl., 24 (1968) 182 - 190.
5. Leonard Gillman and Melvin Henriksen, Rings of continuous functions in which every finitely generated ideal is principal, Trans. Amer. of Math. Soc., 82 (1956) 366 - 391.
6. G.Thangaraj and G.Balasubramanian, On fuzzy basically disconnected space, J.

Fuzzy Math., 9(1) (2001) 103-110.
7. G.Thangaraj and G.Balasubramanian, On somewhat fuzzy continuous functions, J. Fuzzy Math., 11(2) (2003) 725 - 736.
8. G.Thangaraj and K.Dinakaran, On Fuzzy Pseudo - Continuous functions, IOSR J. of Math., 15(5) (2017) 12 - 20.
9. G.Thangaraj and S.Muruganantham, On fuzzy perfectly disconnected spaces, Inter. Jour. of Advances in Math., 2017(5) (2017) 12 - 21.
10. G.Thangaraj and S.Muruganantham, On fuzzy globally disconnected spaces, (Communicated to Tripura Math., Soc., India).
11. G.Thangaraj and R.Palani, On fuzzy weakly Baire spaces, Bull. of Int. Math. Vir. ins., 7 (2017) 476 - 489.
12. G.Thangaraj and E.Poongothai, On fuzzy σ - Baire spaces, Int. J .Fuzzy Math.Sys. , 3(4) (2013) 275 - 283.
13. L.A.Zadeh, Fuzzy sets, Inform. and Control, 8 (1965) 338 - 353.