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Essential Domination of Some Special Graphs

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ABSTRACT

This paper deals with the simple connected graphs which have an essential domination number and unique dominating set.

Keywords: Domination, Essential domination, Dutch windmill graph, Pan graph, Complete graph, wheel graph, fan graph, lollipop graph, star graph.

Mathematical Subject Classification (2010): 05C75

1. Introduction

In this paper a graph $G=(V,X)$, a vertex u of a simple connected graph is an essential dominating vertex, if every minimum dominating set contains the vertex u. The minimum cardinality of the unique dominating set is called the essential domination number. it is denoted by $\gamma_E(G)$. Cockayne and Hedetniemi [1] introduced the concept dominating set. Harary, Norman and Cartwright [2] explained an interesting application in voting situations using the concept of domination. Liu [3] also discussed the application of dominance to communication network, where a dominating set represents a set of cities which acting as transmitting stations, can transmit messages to every city in the network. We also characterized the essential dominating set for the join of one vertex. In this paper we have found out essential domination number and unique domination set of some special graph.

2. Preliminaries

Definition 2.1. Let G be a graph. A subset S of V is called dominating set if every vertex in $V-S$ is adjacent to a vertex in S. The minimum cardinality of a dominating set in G is called the domination number of G and it is denoted by $\gamma(G)$.

Definition 2.2. A vertex u of a simple connected graph is an essential dominating vertex, if every minimum dominating set contains the vertex u. The minimum cardinality of the essential dominating set is called the essential domination number, it is denoted by $\gamma_E(G)$.

Definition 2.3. Dutch windmill graph: the dutch windmill graph $D_m^{(n)}$ is the graph obtained by taking n copies of the cycle C_m with vertex in common. The minimum cardinality of a dominating set in G is called the domination number of G and it is denoted by $\gamma\,(D_m^{(n)})$.

Theorem 2.1. A dutch windmill graph $D_m^{(n)}$ has a essential dominating vertex set, if $n \equiv 0 \left(mod 3 \right)$.

Proof: Let G be a dutch windmill graph with n vertices. And $\Delta = degv$, so must be in the minimal dominating set. $D_m^{(n)} - \{v\}$ becomes a disconnected graph of nP_{m-1}. v is adjacent to every pendent vertices of P_{m-1} in the graph $D_m^{(n)} - \{v\}$. Hence it is enough to dominate the remaining nP_{3m-3} paths. it has unique dominating set S with minimum cardinality $\gamma\,(D_m^{(n)})$ then every element in S is an essential.

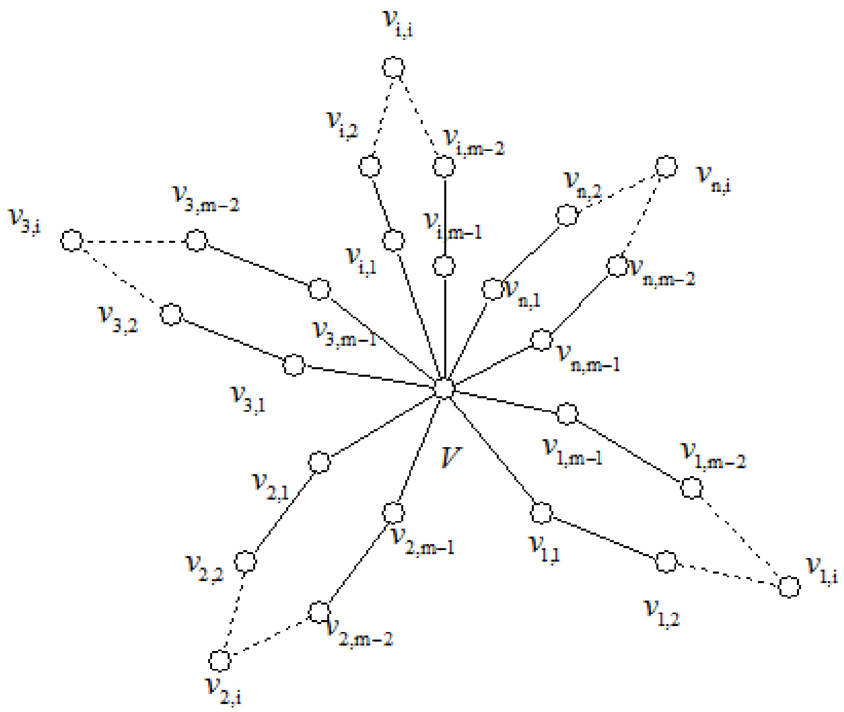

Figure 1:

Theorem 2.2. If G is P_{3n} then G has dominating vertices which are essential.

Proof: P_{3n} has only one dominating set S with minimum cardinality $\gamma(G)$ then every element in S is an essential vertex.

Hence $\{v_2,\ v_5,\ v_8,\ ....v_{3n-1}\}$ is an essential domination set.

Corollary 2.1. For any P_{3n+1} and P_{3n+2} has no essential dominating set.

Proof: A graph P_{3n+1} has more than one minimal dominating set, let $S_1, S_2, \ldots\ldots$ then $S_1 \cap S_2 \cap \ldots\ldots = \phi$ hence P_{3n+1} has no essential vertex.

$S_1 \cap S_2 \cap S_3 \cap \ldots\ldots\ldots = \{v_1\}$ has more than one minimal dominating set, let $S_1, S_{2,}\ldots\ldots\ldots$ But

$V_{3n} \notin S$ and $S_1 \cap S_2 \cap \ldots\ldots \neq \phi$ hence P_{3n+2} has one essential vertex.

Theorem 2.3. A pan graph $\mathbf{P}_n$ has a essential dominating vertex, if $n \not\equiv 1(mod 3)$.

Proof:

Case1: $n \equiv 1(mod 3)$

Let G be a pan graph, it has more than one domination set exist and all vertices have a chance to come in domination set. Let $S_1, S_2, S_3 \ldots\ldots\ldots\ldots$ where $S_1 = \{v_1, v_4, v_{7,}\ldots\ldots\ldots v_{i-3}, v_i\}$ & $S_2 = \{v, v_3, v_{6\ldots\ldots\ldots,} v_{i-4}, v_{i-1}\}$ and so on $S_1 \cup S_2 \cup S_3 \cup S_4 \cup \ldots\ldots\ldots\ldots\ldots = |V(G)|$ but $S_1 \cap S_2 \cap S_3 \cap \ldots\ldots\ldots\ldots = \phi$ so it has no essential vertex.

Case 2: $n \equiv 2(mod 3)$

Let G be a pan graph, it has more than one dominating set exist it has1 essential vertex which node is adjacent to only pendent vertex. That is $S_1 \cap S_2 \cap S_3 \cap \ldots\ldots\ldots\ldots = \{v_1\}$.

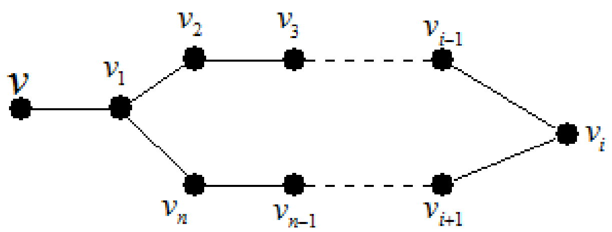

Figure 2:

Case 3: $n \equiv 0(mod 3)$

Let G be a pan graph, it has unique dominating set S with minimum cardinality $\gamma(G)$ then every element in S is an essential, where $S = \{\mathbf{v}_{3k-2;} k = 1, 2, 3\ldots\ldots\}$

Theorem 2.4. If $|V(G)| = n$ and $degv = n-1$ with $deg\, w < n-1$ for all $w \in V(G) - \{v\}$ then v is essential.

Proof: Let $v \in V(G)$ such that $degv = n-1$, clearly $\{v\}$ is the only minimal dominating set of G , Therefore $\{v\}$ is an essential vertex.

Theorem 2.5. For any graph, if $H = G + u$ then u is essential in $\overline{H}$.

Proof: Let G be any graph, its may have more then one components. Let $H = G + u$, u is adjacent to all vertices in G So, u has a maximum degree but u is not necessary essential vertex in H because more then one maximum vertex exist in H . But in $\overline{H}$, u is isolated so u must be exist in minimum dominating set. Hence u is on essential vertex in $\overline{H}$

Theorem 2.6. Let G be a graph with $\gamma(G) = 1$.

Let $S = \{v \in V(G) / N_G[v] = V(G)\}$ then S is essential in $\overline{G}$.

Proof: If $S = \{v \in V(G) / N_G[v] = V(G)\}$ then $degv = 0$ in $\overline{G}$ that is v is an isolated vertex. therefore it must be in every dominating set, hence v is essential vertex thus any isolated vertex thus any isolated vertex is an essential vertex therefore S is essential set in $\overline{G}$.

Theorem 2.7. Let G be any graph and $u \in G$, if u has atleast two pendent edges then u is essential dominating vertex.

Proof: Let G be any graph, $u \in V(G)$ and atleast two pendent edges. u is must contain in $\{S\}$. ie, $u \subset \{S\}$ suppose, $u \not\subset \{S\}$ then there must exist another vertex , which dominates all the vertices adjacent to u. But obviously no such vertex exist. Hence u is essential dominating vertex in G .

Theorem 2.8. Let G be a K_n graph and $v \in G$. Let H be a graph obtained from G by adding any vertex u and adjacent to v then $\overline{H}$ has essential domination set.

Proof: Let G be a K_n graph, in K_n all vertices has degree $n-1$ therefore there exist no essential vertex. Let H be a graph obtained from G by adding any vertex u And adjacent to v now H has $n+1$ vertices and $degv$ is n and $degu$ is one all other vertices has degree $n-1$ clearly v is the only minimal dominating set of H. Therefore v is an essential vertex. In $\overline{H}$ has disconnected graph (only two component) first component $degv$ is zero ie, v is isolated vertex therefore v is essential dominating vertex. And another one is star graph $degu = n-1$ ie, u is essential dominating vertex, since $\overline{H}$ has essential dominating set.

3. The number of essential dominating vertices in the given graph is the essential dominating set.

WHEEL GRAPH: (w_n)

$\gamma(w_3)=1$ has more than one minimal dominating set hence w_3 has no essential vertex.

- Wheel graph (w_n), $\gamma_E(w_n)=1$. where $n>3$ only one minimal dominating set $\{w\}$. and hence w is essential

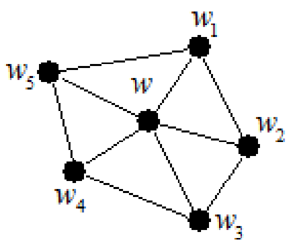

Figure 3:

- STAR GRAPH(S_n), $\gamma_E(S_n)=1$.

For $n>1$, S_n have only one minimal dominating set $\{v\}$. Hence v is essential vertex.

- FAN GRAPH F_n,

F_2, F_3 no essential vertex

F_n $n>3, v$ is essential vertex. Example for F_5

$\overline{K_n}$ Graph , $\gamma_E(\overline{K_n})=n.$

Lollipop graph

The (m,n) lollipop graph is a graph obtained by joining a complete graph K_m to a path P_n on n vertices and its denoted by $L_{m,n}$. In a lollipop graph with the path P_n where $n = 3n-2$ has essential dominating set. Like that, Bi-star, friendship graph, flower graph, windmill graph, P_{3n} ...theses all are having essential domination set.

Result 3.1. Let G be a $K_{m,n}$ graph and $v\in G$. Let H be a graph obtained from G by adding any vertex u and adjacent to v. if $m=n$ then u is essential in $\overline{H}$. And then H has no essential domination vertex. And if $m\neq n$, Let H be a graph obtained from G by adding any vertex u and adjacent to $\delta(u)$ then u is essential in H. But $\overline{H}$ has no essential vertex.

4. Conclusion

In this article, we have discussed few special graphs for which the domination is

Essential. Further research can be done in exploring various graphs with the same property

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