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Multi Objective Multi Item Fuzzy Transportation Problem with Congestion Charge

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ABSTRACT

In the present scenario supply, demand and transportation cost per unit of the commodity in multi objective transportation problem for multi items may be rarely specified precisely due to uncertain conditions. Vehicular emission generates harmful air pollutants in urban areas which leads to severe health hazardous. In this paper, a multi objective profit transportation problem for multi items has been framed and solved in fuzzy environment, it emphasizes on fuzzy methodology to solve transportation problem to minimize the travel time and maximize the profit by reducing emission charge during traffic congestion. Unit transportation cost, environmental protection cost, transportation time and congestion charge during peak time and non peak time, loading and unloading time, total supply and demand, selling and purchasing prices are all considered as triangular type-2 fuzzy numbers. Then the multi objective profit transportation problem has been transformed into single objective transportation problem by applying Fuzzy Goal programming Technique (FGPT) and Weighted Fuzzy Goal Programming Technique (WFGPT) and the corresponding model is solved using Generalized Reduced Gradient (GRG) method LINGO-(18.0). Numerical illustration has been given to show the efficiency of the proposed model.

Keywords: Type-2 triangular fuzzy variable, CV reduction methods, Congestion charge, Profit transportation problem, Fuzzy Goal Programming Technique, Weighted Fuzzy Goal Programming Technique

Mathematical Subject Classification (2010): 90C08

1. Introduction

In current scenario, several establishments in societies have a compulsion to find a superior way to satisfy the needs of the customers in cost effective manner. Transportation problem provides a dynamic structure to face this situation and guarantee the timely shipment of the commodity satisfying the needs of the customers. The classical transportation problem is a particular case of linear programming problem which deals with the dispersion of the commodities from source to destination. The Transportation problem was originally developed by Hitchcock [11] in 1941 and solution to this problem is derived using simplex method. The transportation problem can be modeled as a standard linear programming problem which can then be solved by the simplex method. Suppose that the company wishes to judge the transportation plan in advance for the next month, they need to ask for some experts knowledge or to consider the statistical analysis of previous transportation activities. Owing to the current complex environmental conditions during transportation activities some parameters are treated as uncertain variables to meet the current situation. Saad and Abbas discussed the solutions algorithm for solving transportation problem in fuzzy environment [17]. Chanas et al proposed a fuzzy linear programming model for solving transportation problem with crisp cost coefficient and fuzzy supply and demand values [4]. The tool to manage this imprecise conditions fuzzy methodology considering fuzzy demand and fuzzy supply constraints involving fuzzy triangular variable is used. Chanas and Kuchta [6] proposed the concept of the optimal solution for the transportation problem with fuzzy coefficients expressed as fuzzy numbers and developed an algorithm for obtaining the optimal solution. Zimmermann [22] in 1978 applied fuzzy set theory concept with some membership function to solve multi-objective transportation problem. In 1975 the concept of fuzzy set called as type-2 fuzzy set was introduced by Zadeh [21] and the extension of ordinary fuzzy set called as type-2 fuzzy set was introduced by Zadeh [19] in 1975. Few Reduction method is used to convert type-2 fuzzy variable to type-1 fuzzy variable and then defuzzification is carried out using the concept of centroid method or by using geometric defuzzification method. Karnik and Mendel [13] introduced a method for type reduction via the concept of a centroid of a type-2 fuzzy set. In 2011 Qin et al [16] introduced three kinds of critical value reduction methods called as optimistic critical value, pessimistic critical value and critical value of type-2 fuzzy variables and the expected value for type-2 triangular fuzzy variable was established. Bellmann and Zadeh suggested fuzzy programming model to make decision in fuzzy environment [2]. Fuzzy transportation problem is the problem of minimizing fuzzy valued objective function with supply and demand. The Goal programming technique and weighted goal programming technique was first formulated by Charnes and Cooper in 1990 [3]. In order to measure a fuzzy event, Zadeh [20] defined a concept of possibility measure as a counterpart of probability measure in 1978. Then the possibility measure was studied by Klir [12], Dubois and Prode [8]. Liu Liu [14] proposed credibility measure. Oheigeartaigh [15] proposed an algorithm for solving transportation problems in which the capacities and requirements are fuzzy sets with triangular membership functions. In 2017, Dutta and Jana [7] formulated the expectations of the reductions of type-2 trapezoidal fuzzy variables and applied it to a multi objective solid transportation problem via goal programming technique. In 2017, D.K Jana et al [7] proposed a comparative study on credibility measures of type-1 and type-2 fuzzy variables and applied it to multi objective profit transportation problem via goal programming technique.

Traffic congestion is the vital problem during transportation activity. It indirectly contributes to the environmental pollution because of the emission of GHG gases from the vehicles which in turn cause harmful damages to the ozone layer. It also contributes to many other negative effects such as more fuel expenditure, delay in delivery of goods and also causes severe health hazardous to the travelers. In order to reduce these congestion related problem during transportation activity, the congestion cost is charged on a particular route in which the transportation process is carried out. The congestion charge varies during peak time (i.e., after 8 a.m. and before 8 p.m.) will have a charge doubled than the normal charge and non-peak time will have normal charge (i.e., before 8 a.m. and after 8 p.m.). This type of toll charging is already in practice in many countries [18].

If this strategy is implemented in India, the negative impact of traffic congestion gets much reduced. Therefore encourages the timely shipment of commodities from source to destination and drastically reduces the emission of chemical pollutants form vehicles which in turn increases the profit and the time is minimized during transportation process. In section-2, the preliminaries of Type-2 Fuzzy set have been discussed. In section-3 the Fuzzy Goal Programming Technique (FGPT) and Weighted Goal Programming Technique (WFGPT) have been explained. In section-4, the mathematical notations and assumptions used in this paper has been given. In section-5 the formulation of Type-2 Fuzzy Multi Objective Multi Items Profit Transportation Problem (T2FMOMIPTP) with congestion charge. Section-6 detailed the solution procedure for T2FMOMIPTP. Section-7 discusses the numerical example for the proposed T2FMOMIPTP model. Section-8 gives the conclusion.

2. Preliminaries

In this section, some basic definition on T2 fuzzy sets have been discussed.

Definition 1. Type 2 triangular fuzzy variable [7]

A type-2 triangular fuzzy variable $\tilde{\tilde{\xi}}$ is denoted by $\tilde{\tilde{\xi}} = (r_1, r_2, r_3, \theta_r, \theta_l)$ where r_1, r_2, r_3 are real values and θ_r, $\theta_l \in [0, 1]$ are two parameters characterizing the degree of uncertainty that ξ takes a value in R. For $x \in [r_1, r_2]$ the secondary possibility function $\mu_{\tilde{\xi}}(x)$ of $\tilde{\xi}$ is defined in the form

$$\mu_{\tilde{\xi}}(x) = \left(\frac{x-r_1}{r_2-r_1} + \theta_l \min\left\{ \frac{x-r_1}{r_2-r_1} - \frac{r_2-x}{r_2-r_1} \right\}, \frac{x-r_1}{r_2-r_1}, \frac{x-r_1}{r_2-r_1} + \theta_r \min\left\{ \frac{x-r_1}{r_2-r_1}, \frac{x-r_1}{r_2-r_1} \right\} \right)$$ F

or $x \in [r_1, r_2)$, the secondary possibility distribution function $\tilde{\mu}_{\tilde{\tilde{\xi}}}$,

$$\tilde{\mu}_{\tilde{\tilde{\xi}}} = \left(\frac{r_3-x}{r_3-r_2} + \theta_l \min\left\{ \frac{r_3-x}{r_3-r_2} - \frac{x-r_2}{r_3-r_2} \right\}, \frac{r_3-x}{r_3-r_2}, \frac{r_3-x}{r_3-r_2} + \theta_r \min\left\{ \frac{r_3-x}{r_3-r_2}, \frac{x-r_2}{r_3-r_2} \right\} \right)$$

Definition 2. Type-2 Fuzzy set ([10])

Let x X be a Universal set. Then a type-2 fuzzy set denoted by $\tilde{\tilde{A}}$ is characterized by a type-2 membership function of the form $\tilde{\mu}_{\tilde{\tilde{A}}}(x,u)$, where $x \in X$ and J_x denoting the primary membership of x such that $u \in J_x \subseteq [0, 1]$,

$$\tilde{\tilde{A}} = \{((x,u), \tilde{\mu}_{\tilde{\tilde{A}}}(x,u)) \mid \forall x \in X, \forall u \in J_x \subseteq [0,1]\}, \quad \text{in which} \quad 0 \le \tilde{\mu}_{\tilde{\tilde{A}}}(x,u) \le 1.$$

$\tilde{\tilde{A}}$ can be expressed as $\tilde{\tilde{A}} = \int_{x \in X} \int_{u \in J_x} \tilde{\mu}_{\tilde{\tilde{A}}}(x,u)/(x,u), J_x \in [0,1]$, where ∫ ∫ denotes the union of x and u.

3. Fuzzy methodology

Method 1. Fuzzy Goal Programming Technique [7]

The fuzzy Goal Programming Technique (FGPT) is introduced to solve linear and non-linear multi objective programming problems (MOPPs) by converting them into single objective optimization problems.

The MOPPs can be considered with m-objective functions $f_{i,}$ i = 1, 2,....m and may be written as $\{\min[f_1(x), f_2(x), \ldots f_m(x)]$

Let us consider that decision makers have fixed the membership function $\mu_k(f_k(x))$ and given the goal membership function value (k = 1, 2,....).

Let us consider the following programming problem as

$$\min \sum_{i=1}^{m} d_i^-$$

subject to the constraints $\mu_k(f_k(x)) + d_i^+ - d_i^- = \bar{\mu}_k$

$d_i^+ . d_i^- = 0, d_i^+ . d_i^- \geq= 0, k = 0, 1, 2, \ldots .m$

$d_i^+ \geq 0, d_i^- \geq 0,$ where d_i^+, d_i^- denotes the positive and negative deviations respectively.

Method 2. Weighted Fuzzy Goal Programming Technique [7]

The Weighted Goal Programming Technique (WFGPT) is introduced to solve linear and non-linear multi objective programming problems (MOPPS) by converting them into single objective optimization problems.

The MOPPs can be considered with m-objective functions $f_{i,}$ i = 1, 2, ...m and may be written as $\{\min[f_1(x), f_2(x), \ldots f_m(x)]$

Let us consider that decision makers have fixed the membership function $\mu_k(f_k(x))$ and given the weighted goal membership function value (k = 1, 2,....m).

Let us consider the following programming problem as

$$\min \sum_{i=1}^{m} w(d_i^- + d_i^+)$$

subject to the constraints $w_k(f_k(x)) + w_i^+ d_i^+ - w_i^- d_i^- = \bar{w}k$

$d_i^+ . d_i^- = 0, d_i^+ . d_i^- \geq 0, k = 0, 1, 2, \ldots .m$

$d_i^+ \geq 0, d_i^- \geq 0,$ where d_i^+, d_i^- denotes the positive and negative deviations respectively.

4. Notations and assumptions

This section defines the mathematical notations and assumptions used in this paper. The following notations are used in this paper.

W-number of sources (indexed i = 1, 2,...W), F-number of destinations (indexed j = 1, 2, ...F) I-number of items (indexed k = 1, 2,....I) $\tilde{\tilde{e}}_{ij}$-total environmental protection cost $\tilde{\tilde{a}}_{ik}$-supply (in litres) $\tilde{\tilde{b}}_{jk}$-demand (in litres) $\tilde{\tilde{c}}_{ijk}$-unit transportation cost from ith source to jth destination (in paise/litre), $(a)\,(\tilde{\tilde{pt}})_{ijk}$ -unit transportation time from ith source to jth destination during peak time (in minutes), (b) $(\tilde{\tilde{npt}})_{ijk}$ -unit transportation time from ith source to jth destination during non-peak time (in minutes), $\tilde{\tilde{d}}_{ijk}$ -loading and unloading time from ith source to jth destination, x_{ijk}-the amount (in litres) to be transported for ith source to jth destination, $\tilde{\tilde{s}}_{jk}$ -the selling price of the product at jth destination (in paise/litre), $\tilde{\tilde{p}}_{ik}$ -the purchasing price of the product at ith source (in paise/litres). (a) $(\tilde{\tilde{pc}})_{ijk}$ -congestion charge during peak time, (b) $(\tilde{\tilde{npc}})_{ijk}$ -congestion charge during non-peak time, $\tilde{\tilde{f}}_1$ -total profit in the problem (in paise), $\tilde{\tilde{f}}_2$ -total transportation time (in minutes)

4.1. Assumptions

In the T2FPMOMITP the following assumption is made.

If a commodity of the company is to be transported to different destination then the company has to pay the environmental protection cost. Hence the following binary indicator is introduced as

$$y_{ijk} = \begin{cases} i, & \text{if} \quad x_{ijk} \neq 0 \\ 0, & \text{if} \quad x_{ijk} = 0 \end{cases} \tag{1}$$

5. Formation of T2FPMOMITP

In this proposed model, the following conditions are optimized separately during peak time and non-speak time.

(i) Maximize the total profit (ii) Minimize the total transportation time

The T2FPTP is formulated as

$$\max \tilde{\tilde{f}}_1 = \sum_{i=1}^{W}\sum_{j=1}^{F}\sum_{k=1}^{I}\left\{(\tilde{\tilde{s}}_{jk} - \tilde{\tilde{p}}_{ik} - \tilde{\tilde{c}}_{ijk} - \tilde{\tilde{pc}}_{ijk})x_{ijk} - \tilde{\tilde{e}}_{ijk} \times y_{ijk}\right\} \tag{2}$$

$$\min \tilde{\tilde{f}}_2 = \sum_{i=1}^{W}\sum_{j=1}^{F}\sum_{k=1}^{I}\left\{\tilde{\tilde{pt}}_{ijk}\,.y_{ijk} + \tilde{\tilde{d}}_{ijk}\,.x_{ijk}\right\} \tag{3}$$

subject to the constraints:

$$\sum_{j=1}^{F}\sum_{k=1}^{I} x_{ijk} \leq \tilde{\tilde{a}}_{ik}, \forall i, k \tag{4}$$

$$\sum_{i=1}^{W}\sum_{k=1}^{I} x_{ijk} \leq \tilde{\tilde{b}}_{jk}, \forall j, k \tag{5}$$

$$x_{ijk} \geq 0 \tag{6}$$

Similarly, the T2FPTP can also be formulated for non-peak time.

Suppose the commodity is to be transported to a destination and no expert knowledge about the transportation cost, supply and demand. This uncertainty is examined by considering type-2 triangular fuzzy cost. The objective functions $\tilde{\tilde{f}}_i (i = 1, 2, 3, 4)$ consists of selling price $(\tilde{\tilde{s}})_{jk} = (s_{jk}^1, s_{jk}^2, s_{jk}^3, s_{jk}^{\theta_l}, s_{jk}^{\theta_r})$,

purchasing price $(\tilde{\tilde{p}})_{ik} = (p_{ik}^1, p_{ik}^2, p_{ik}^3, p_{ik}^{\theta_l}, p_{ik}^{\theta_r})$,

transportation cost $(\tilde{\tilde{c}})_{ijk} = (c_{ijk}^1, c_{ijk}^2, c_{ijk}^3, c_{ijk}^{\theta_t}, c_{ijk}^{\theta_r})$,

total environment protection cost $\tilde{\tilde{e}}_{ijk} = (e_{ijk}^1, e_{ijk}^2, e_{ijk}^3, e_{ijk}^{\theta_t}, e_{ijk}^{\theta_r})$,

loading time and unloading time $(\tilde{\tilde{d}})_{ijk} = (d_{ijk}^1, d_{ijk}^2, d_{ijk}^3, d_{ijk}^{\theta_t}, d_{ijk}^{\theta_r})$,

congestion charge during peak time $(\tilde{\tilde{pc}})_{ijk} = ((pc)_{ijk}^1, (pc)_{ijk}^2, (pc)_{ijk}^3, (pc)_{ijk}^{\theta_t}, (pc)_{ijk}^{\theta_r})$,

congestion charge during non-peak time

$(\tilde{\tilde{npc}})_{ijk} = ((npc)_{ijk}^1, (npc)_{ijk}^2, (npc)_{ijk}^3, (npc)_{ijk}^{\theta_t}, (npc)_{ijk}^{\theta_r})$,

transportation time during non-peak time

$(\tilde{\tilde{nt}})_{ijk} = ((nt)_{ijk}^1, (nt)_{ijk}^2, (nt)_{ijk}^3, (nt)_{ijk}^{\theta_t}, (nt)_{ijk}^{\theta_r})$,

transportation time during peak time $(\tilde{\tilde{pt}})_{ijk} = ((pt)_{ijk}^1, (pt)_{ijk}^2, (pt)_{ijk}^3, (pt)_{ijk}^{\theta_t}, (pt)_{ijk}^{\theta_r})$,

supply $\tilde{\tilde{a}}_{ik} = (a_{ik}^1, a_{ik}^2, a_{ik}^3, a_{ik}^{\theta_t}, a_i^{\theta_r})$, demand $\tilde{\tilde{b}}_{jk} = (b_{jk}^1, b_{jk}^2, b_{jk}^3, b_{jk}^{\theta_t}, b_{jk}^{\theta_r})$.

6. Solution procedure for T2FPMOMITP

The Type-2 fuzzy variable can be reduced to non fuzzy variable using critical value reduction methods (CV).

Special cases of CV methods are discussed as follows:

6.1. Optimistic expected value

Using the generalized expectation theory [5], the objective functions and the constructions for T2FPMOMITP can be written as:

$$\max f_1 = \sum_{i=1}^{W} \sum_{j=1}^{F} \sum_{k=1}^{I} \left[\left\{ \frac{s_{jk}^1 + s_{jk}^3}{2} - \frac{(s_{jk}^1 - 2s_{jk}^2 + s_{jk}^3) \ln(1 + \frac{s_{jk}^{\theta_r}}{2})}{2s_{jk}^{\theta_r}} \right. \right.$$

$$-\frac{p_{jk}^{1}+p_{jk}^{3}}{2}-\frac{(p_{ik}^{1}-2p_{ik}^{2}+p_{ik}^{3})\ln(1+\frac{p_{ik}^{\theta_r}}{2})}{2p_{ik}^{\theta_r}}$$

$$-\frac{c_{ijk}^{1}+c_{ijk}^{3}}{2}-\frac{(c_{ijk}^{1}-2c_{ijk}^{2}+c_{ijk}^{3})\ln(1+\frac{c_{ijk}^{\theta_r}}{2})}{2c_{ijk}^{\theta_r}}-\frac{(pc)_{ijk}^{1}+(pc)_{ijk}^{3}}{2}$$

$$\left.-\frac{((pc)_{ijk}^{1}-2(pc)_{ijk}^{2}+(pc)_{ijk}^{3})\ln(1+\frac{(pc)_{ijk}^{\theta_r}}{2}}{2(pc)_{ijk}^{\theta_r}}\right\}x_{jik}$$

$$\left.-\frac{e_{ijk}^{1}+e_{ijk}^{3}}{2}-\frac{(e_{ijk}^{1}-2e_{ijk}^{2}+e_{ijk}^{3})\ln(1+\frac{e_{ijk}^{\theta_r}}{2})}{2e_{ijk}^{\theta_r}}\times y_{ijk}\right] \tag{7}$$

$$\min f_2=\sum_{i=1}^{W}\sum_{j=1}^{F}\sum_{k=1}^{I}\left[\left[\left\{\frac{(pt)_{ijk}^{1}+(pt)_{ijk}^{3}}{2}-\frac{((pt)_{ijk}^{1}-2(pt)_{ijk}^{2}+(pt)_{ijk}^{3})\ln(1+\frac{(pt)_{ijk}^{\theta_r}}{2}}{2(pt)_{ijk}^{\theta_r}}\right\}y_{ijk}\right.\right.$$

$$\left.+\left\{\frac{d_{ijk}^{1}+d_{ijk}^{3}}{2}-\frac{(d_{ijk}^{1}-2d_{ijk}^{2}+d_{ijk}^{3})\ln(1+\frac{d_{ijk}^{\theta_r}}{2})}{2d_{ijk}^{\theta_r}}\right\}x_{ijk}\right] \tag{8}$$

Subject to the constraints

$$\sum_{j=1}^{F}\sum_{k=1}^{I}x_{ijk}\le\frac{a_{ik}^{1}+a_{ik}^{3}}{2}-\frac{(a_{ik}^{1}-2a_{ik}^{2}+a_{ik}^{3})\ln(1+\frac{a_{ik}^{\theta_r}}{2})}{2a_{ik}^{\theta_r}},\forall i,k \tag{9}$$

$$\sum_{i=1}^{W}\sum_{k=1}^{I}x_{ijk}\le\frac{b_{jk}^{1}+b_{jk}^{3}}{2}-\frac{(b_{jk}^{1}-2b_{jk}^{2}+b_{jk}^{3})\ln(1+\frac{b_{jk}^{\theta_r}}{2})}{2b_{jk}^{\theta_r}},\forall j,k \tag{10}$$

Similarly the objective function f_3 and f_4, their corresponding constraints for the proposed T2FPMOMITP during non-peak time can also be formulated.

6.2. Pessimistic expected value

Using the generalized expectation theory [5], the objective functions and the constraints for T2FPMOMITP can be written as:

$$\max\ f_1 = \sum_{i=1}^{W}\sum_{j=1}^{F}\sum_{k=1}^{I}\left[\left\{ s_2^j - \frac{(s_j^1 - 2s_j^2 + s_j^3)\ln(1+\frac{s_j^{\theta_r}}{2})}{2s_j^{\theta_r}}\right.\right.$$

$$-p_j^2 - \frac{(p_j^1 - 2p_i^2 + p_i^3)\ln(1+\frac{p_i^{\theta_r}}{2})}{2p_i^{\theta_r}} - c_{ijk}^2 - \frac{(c_{ijk}^1 - 2c_{ijk}^2 + c_{ijk}^3)\ln(1+\frac{c_{ijk}^{\theta_r}}{2})}{2c_{ijk}^{\theta_r}} - (pc)_{ijk}^2$$

$$\left. - \frac{(pc)_{ijk}^1 - 2(pc)_{ijk}^2 + (pc)_{ijk}^3)\ln(1+\frac{(pc)_{ijk}^{\theta_r}}{2})}{2(pc)_{ijk}^{\theta_r}}\right\} x_{ijk}$$

$$\left. -e_{ijk}^2 - \frac{(e_1^{ijk} - 2e_{ijk}^2 + e_{ijk}^3)\ln(1+\frac{e_{ijk}^{\theta_r}}{2})}{2e_{ijk}^{\theta_r}} \times y_{ijk}\right] \tag{11}$$

$$\min\ f_2 = \sum_{i=1}^{W}\sum_{j=1}^{F}\sum_{k=1}^{I}\left[\left\{(pt)_{ijk}^2 - \frac{((pt)_{ijk}^1 - 2(pt)_{ijk}^2 + (pt)_{ijk}^3)\ln(1+\frac{(pt)_{ijk}^{\theta_r}}{2})}{2(pt)_{ijk}^{\theta_r}}\right\} y_{ijk}\right.$$

$$\left. + \left\{ d_{ijk}^2 - \frac{(d_{ijk}^1 - 2d_{ijk}^2 + d_{ijk}^3)\ln(1+\frac{d_{ijk}^{\theta_r}}{2})}{2d_{ijk}^{\theta_r}}\right\} x_{ijk}\right] \tag{12}$$

and with subject to the constraints (9) and (10).

Similarly the objective function f_3 and f_4, their corresponding constraints for the proposed T2FPMOMITP during non-peak time can also be formulated.

6.3. Expected critical value

Using the generalized expectation theory [5], the objective functions and the constraints for T2FPMOMITP can be written as:

$$\max\ f_1 = \sum_{i=1}^{W}\sum_{j=1}^{F}\sum_{k=1}^{I}\left[\left\{\frac{s_{jk}^1 + 2s_{jk}^2 + s_{jk}^3}{4} - \frac{s_{jk}^1 + 2s_{jk}^2 + s_{jk}^3}{8}\right.\right.$$

$$\left\{\frac{1}{s_{jk}^{\theta_r}} - \frac{1}{s_{jk}^{\theta_l}} - \frac{(1+s_{jk}^{\theta_r})\ln(1+s_{jk}^{\theta_r})}{s_{jk}^{\theta_r^2}} + \frac{(1+s_{jk}^{\theta_l})\ln(1+s_{jk}^{\theta_l})}{s_{jk}^{\theta_l^2}}\right\}$$

$$-\frac{p_{jk}^{1}+2p_{jk}^{2}+p_{jk}^{3}}{4}+\frac{p_{jk}^{1}+2p_{jk}^{2}+p_{jk}^{3}}{8}$$

$$\left\{\frac{1}{p_{jk}^{\theta_r}}-\frac{1}{p_{jk}^{\theta_l}}-\frac{(1+p_{jk}^{\theta_r})\ln(1+p_{jk}^{\theta_r})}{p_{jk}^{\theta_r^2}}+\frac{(1+p_{jk}^{\theta_l})\ln(1+p_{jk}^{\theta_l})}{p_{jk}^{\theta_l^2}}\right\}$$

$$-\frac{c_{ijk}^{1}+2c_{ijk}^{2}+c_{ijk}^{3}}{4}+\frac{c_{ijk}^{1}+2c_{ijk}^{2}+c_{ijk}^{3}}{8}$$

$$\left\{\frac{1}{c_{ijk}^{\theta_r}}-\frac{1}{c_{ijk}^{\theta_l}}-\frac{(1+c_{ijk}^{\theta_r})\ln(1+c_{ijk}^{\theta_r})}{c_{ijk}^{\theta_r^2}}+\frac{(1+c_{ijk}^{\theta_l})\ln(1+c_{ijk}^{\theta_l})}{c_{ijk}^{\theta_l^2}}\right\}$$

$$-\frac{(pc)_{ijk}^{1}+2(pc)_{ijk}^{2}+(pc)_{ijk}^{3}}{4}+\frac{(pc)_{ijk}^{1}+2(pc)_{ijk}^{2}+(pc)_{ijk}^{3}}{8}$$

$$\left\{\frac{1}{(pc)_{ijk}^{\theta_r}}-\frac{1}{(pc)_{ijk}^{\theta_l}}-\frac{(1+(pc)_{ijk}^{\theta_r}\ln(1+(pc)_{ijk}^{\theta_r}}{(pc)_{ijk}^{\theta_r^2}}\right.$$

$$\left.\left.+\frac{(1+(pc)_{ijk}^{\theta_t})\ln(1+(pc)_{ijk}^{\theta_l}}{(pc)_{ijk}^{\theta_l^2}}\right\}\right\}x_{ijk}$$

$$-\frac{e_{ijk}^{1}+2e_{ijk}^{2}+e_{ijk}^{3}}{4}+\frac{e_{ijk}^{1}+2e_{ijk}^{2}+e_{ijk}^{3}}{8}$$

$$\left.\left\{\frac{1}{e_{ijk}^{\theta_r}}-\frac{1}{e_{ijk}^{\theta_l}}-\frac{(1+e_{ijk}^{\theta_r})\ln(1+e_{ijk}^{\theta_r})}{e_{ijk}^{\theta_r^2}}+\frac{(1+e_{ijk}^{2})\ln(1+e_{ijk}^{\theta_l})}{e_{ijk}^{\theta_l^2}}\right\}\times y_{ijk}\right] \tag{13}$$

$$\min f_2=\sum_{i=1}^{W}\sum_{j=1}^{F}\sum_{k=1}^{I}\left[\left\{\frac{(pt)_{ijk}^{1}+2(pt)_{ijk}^{2}+(pt)_{ijk}^{3}}{4}+\frac{(pt)_{ijk}^{1}2(pt)_{ijk}^{2}+(pt)_{ijk}^{3}}{8}\right.\right.$$

$$\left.\left\{\frac{1}{(pt)_{ijk}^{\theta_r}}-\frac{1}{(pt)_{ijk}^{\theta_l}}-\frac{(1+(pt)_{ijk}^{\theta_r})\ln(1+(pt)_{ijk}^{\theta_r}}{(pt)_{ijk}^{\theta_r^2}}+\frac{(1+(pt)_{ijk}^{\theta_l})\ln(1+(pt)_{ijk}^{\theta_l}}{(pt)_{ijk}^{\theta_r^2}}\right\}\right\}y_{ijk}$$

$$+\left\{\frac{d_{ijk}^{1}+2d_{ijk}^{2}+d_{ijk}^{3}}{4}+\frac{d_{ijk}^{1}+2d_{ijk}^{2}+d_{ijk}^{3}}{8}\right.$$

$$\left.\left.\left\{\frac{1}{d_{ijk}^{\theta_r}}-\frac{1}{d_{ijk}^{\theta_l}}-\frac{(1+d_{ijk}^{\theta_r})\ln(1+d_{ijk}^{\theta_r})}{d_{ijk}^{\theta_r^2}}+\frac{(1+d_{ijk}^{\theta_l})\ln(1+d_{ijk}^{\theta_l})}{d_{ijk}^{\theta_l^2}}\right\}\right\}x_{ijk}\right] \tag{14}$$

and with subject to the constraints (9) & (10). Similarly the objective function f_3 and f_4 with their corresponding constraints for the proposed T2FPMOMITP during non-peak time can also be formulated.

7. Case study

7.1. Input data

In order to examine the validity of the proposed model a case study is done in milk centres functioning in Trichy and Thanjavur districts based on the secondary data received through RTI. Considering these two as sources (W = 2) and five receiving milk centres from these sources as destinations (F = 5), the two milk varieties such as Standardized milk and Double toned milk is transported for daily sales. Owing to the real life uncertain situations, the transportation problem cannot be analyzed using the crisp value so that the parameters used in the transportation problem are considered as type-2 fuzzy variable. The type-2 fuzzy transportation cost, total environment protection cost, congestion charge during peak-time and non-peak time and transportation time for a unit quantity of the commodity from the i^{th} source to the j^{th} destination and the selling prices and purchasing prices, the total supply in each source and total demand in each destination, for multi items (I = 2) have been assumed and the values are given in the following table.

Table 1.1: Input data for T2FPMOMITP

Item-1	Item-2
$\overset{\approx}{s}_{11} = (4000, 4102, 4102, 0.5, 0.5)$,	$\overset{\approx}{s}_{12} = (4400, 4452, 4552, 0.5, 0.5)$
$\overset{\approx}{s}_{21} = (4000, 4102, 4102, 0.5, 0.5)$	$\overset{\approx}{s}_{22} = (4400, 4452, 4552, 0.5, 0.5)$
$\overset{\approx}{p}_{11} = (2700, 2802, 2852, 0.5, 0.5)$	$\overset{\approx}{p}_{12} = (2700, 2802, 2852, 0.5, 0.5)$
$\overset{\approx}{p}_{21} = (2700, 2802, 2852, 0.5, 0.5)$	$\overset{\approx}{p}_{22} = (2700, 2802, 2852, 0.5, 0.5)$
$\overset{\approx}{a}_{11} = (941, 942, 943, 0.5, 0.5)$	$\overset{\approx}{a}_{12} = (235, 236, 237, 0.5, 0.5)$
$\overset{\approx}{a}_{21} = (225, 226, 227, 0.5, 0.5)$	$\overset{\approx}{a}_{22} = (56, 57, 58, 0.5, 0.5)$
$\overset{\approx}{b}_{11} = (901, 902, 903, 0.5, 0.5)$	$\overset{\approx}{b}_{12} = (224, 225, 226, 0.5, 0.5)$
$\overset{\approx}{b}_{21} = (33, 34, 35, 0.5, 0.5)$	$\overset{\approx}{b}_{22} = (7, 8, 9, 0.5, 0.5)$
$\overset{\approx}{b}_{31} = (5, 6, 7, 0.5, 0.5)$	$\overset{\approx}{b}_{32} = (3, 5, 6, 0.5, 0.5)$
$\overset{\approx}{b}_{41} = (2, 3, 4, 0.5, 0.5)$	$\overset{\approx}{b}_{42} = (4, 7, 8, 0.5, 0.5)$
$\overset{\approx}{b}_{51} = (159, 160, 161, 0.5, 0.5)$	$\overset{\approx}{b}_{52} = (39, 40, 41, 0.5, 0.5)$
$\overset{\approx}{e}_{111} = (22, 23, 24, 0.5, 0.5)$	$\overset{\approx}{e}_{112} = (22, 23, 24, 0.5, 0.5)$
$\overset{\approx}{e}_{121} = (56, 57, 58, 0.5, 0.5)$	$\overset{\approx}{e}_{122} = (56, 57, 58, 0.5, 0.5)$
$\overset{\approx}{e}_{131} = (39, 40, 41, 0.5, 0.5)$	$\overset{\approx}{e}_{132} = (39, 40, 41, 0.5, 0.5)$

$\overset{\approx}{e}_{141} = (58, 59, 60, 0.5, 0.5)$	$\overset{\approx}{e}_{142} = (58, 59, 60, 0.5, 0.5)$
$\overset{\approx}{e}_{151} = (40, 41, 42, 0.5, 0.5)$	$\overset{\approx}{e}_{152} = (40, 41, 42, 0.5, 0.5)$
$\overset{\approx}{e}_{211} = (39, 40, 41, 0.5, 0.5)$	$\overset{\approx}{e}_{212} = (39, 40, 41, 0.5, 0.5)$
$\overset{\approx}{e}_{221} = (89, 90, 91, 0.5, 0.5)$	$\overset{\approx}{e}_{222} = (89, 90, 91, 0.5, 0.5)$
$\overset{\approx}{e}_{231} = (51, 52, 53, 0.5, 0.5)$	$\overset{\approx}{e}_{232} = (51, 52, 53, 0.5, 0.5)$
$\overset{\approx}{e}_{241} = (33, 34, 35, 0.5, 0.5)$	$\overset{\approx}{e}_{242} = (33, 34, 35, 0.5, 0.5)$
$\overset{\approx}{e}_{251} = (39, 40, 41, 0.5, 0.5)$	$\overset{\approx}{e}_{252} = (39, 40, 41, 0.5, 0.5)$
$\overset{\approx}{d}_{111} = (15, 16, 17, 0.5, 0.5)$	$\overset{\approx}{d}_{112} = (15, 16, 17, 0.5, 0.5)$
$\overset{\approx}{d}_{121} = (15, 16, 17, 0.5, 0.5)$	$\overset{\approx}{d}_{112} = (15, 16, 17, 0.5, 0.5)$
$\overset{\approx}{d}_{131} = (15, 16, 17, 0.5, 0.5)$	$\overset{\approx}{d}_{132} = (15, 16, 17, 0.5, 0.5)$
$\overset{\approx}{d}_{141} = (15, 16, 17, 0.5, 0.5)$	$\overset{\approx}{d}_{142} = (15, 16, 17, 0.5, 0.5)$
$\overset{\approx}{d}_{151} = (15, 16, 17, 0.5, 0.5)$	$\overset{\approx}{d}_{152} = (15, 16, 17, 0.5, 0.5)$
$\overset{\approx}{d}_{211} = (15, 16, 17, 0.5, 0.5)$	$\overset{\approx}{d}_{212} = (15, 16, 17, 0.5, 0.5)$
$\overset{\approx}{d}_{221} = (15, 16, 17, 0.5, 0.5)$	$\overset{\approx}{d}_{222} = (15, 16, 17, 0.5, 0.5)$
$\overset{\approx}{d}_{231} = (15, 16, 17, 0.5, 0.5)$	$\overset{\approx}{d}_{232} = (15, 16, 17, 0.5, 0.5)$
$\overset{\approx}{d}_{241} = (15, 16, 17, 0.5, 0.5)$	$\overset{\approx}{d}_{242} = (15, 16, 17, 0.5, 0.5)$
$\overset{\approx}{d}_{251} = (15, 16, 17, 0.5, 0.5)$	$\overset{\approx}{d}_{252} = (15, 16, 17, 0.5, 0.5)$
$\left(\overset{\approx}{pc}\right)_{111} = (0, 0, 0, 0)$	$\left(\overset{\approx}{pc}\right)_{112} = (0, 0, 0, 0)$
$\left(\overset{\approx}{pc}\right)_{121} = (32, 33, 34, 0.5, 0.5)$	$\left(\overset{\approx}{pc}\right)_{122} = (32, 33, 34, 0.5, 0.5)$
$\left(\overset{\approx}{pc}\right)_{131} = (46, 47, 48, 0.5, 0.5)$	$\left(\overset{\approx}{pc}\right)_{132} = (46, 47, 48, 0.5, 0.5)$
$\left(\overset{\approx}{pc}\right)_{141} = (55, 56, 57, 0.5, 0.5)$	$\left(\overset{\approx}{pc}\right)_{142} = (55, 56, 57, 0.5, 0.5)$
$\left(\overset{\approx}{pc}\right)_{151} = (50, 51, 52, 0.5, 0.5)$	$\left(\overset{\approx}{pc}\right)_{152} = (50, 51, 52, 0.5, 0.5)$
$\left(\overset{\approx}{pc}\right)_{211} = (53, 54, 55, 0.5, 0.5)$	$\left(\overset{\approx}{pc}\right)_{212} = (53, 54, 55, 0.5, 0.5)$

$\left(\overset{\approx}{pc}\right)_{221}=\left(30,31,32,0.5,0.5\right)$	$\left(\overset{\approx}{pc}\right)_{222}=\left(30,31,32,0.5,0.5\right)$
$\left(\overset{\approx}{pc}\right)_{231}=\left(0,0,0,0,0\right)$	$\left(\overset{\approx}{pc}\right)_{232}=\left(0,0,0,0,0\right)$
$\left(\overset{\approx}{pc}\right)_{241}=\left(0,0,0,0,0\right)$	$\left(\overset{\approx}{pc}\right)_{242}=\left(0,0,0,0,0\right)$
$\left(\overset{\approx}{pc}\right)_{251}=\left(0,0,0,0,0\right)$	$\left(\overset{\approx}{pc}\right)_{252}=\left(0,0,0,0,0\right)$
$\left(\overset{\approx}{npc}\right)_{111}=\left(0,0,0,0,0\right)$	$\left(\overset{\approx}{npc}\right)_{112}=\left(0,0,0,0,0\right)$
$\left(\overset{\approx}{npc}\right)_{121}=\left(16,17,18,0.5,0.5\right)$	$\left(\overset{\approx}{npc}\right)_{122}=\left(16,17,18,0.5,0.5\right)$
$\left(\overset{\approx}{npc}\right)_{131}=\left(60,61,62,0.5,0.5\right)$	$\left(\overset{\approx}{npc}\right)_{132}=\left(60,61,62,0.5,0.5\right)$
$\left(\overset{\approx}{npc}\right)_{141}=\left(27,28,29,0.5,0.5\right)$	$\left(\overset{\approx}{npc}\right)_{142}=\left(27,28,29,0.5,0.5\right)$
$\left(\overset{\approx}{npc}\right)_{151}=\left(20,21,22,0.5,0.5\right)$	$\left(\overset{\approx}{npc}\right)_{152}=\left(20,21,22,0.5,0.5\right)$
$\left(\overset{\approx}{npc}\right)_{211}=\left(28,29,30,0.5,0.5\right)$	$\left(\overset{\approx}{npc}\right)_{212}=\left(28,29,30,0.5,0.5\right)$
$\left(\overset{\approx}{npc}\right)_{221}=\left(16,17,18,0.5,0.5\right)$	$\left(\overset{\approx}{npc}\right)_{222}=\left(16,17,18,0.5,0.5\right)$
$\left(\overset{\approx}{npc}\right)_{231}=\left(0,0,0,0,0\right)$	$\left(\overset{\approx}{npc}\right)_{232}=\left(0,0,0,0,0\right)$
$\left(\overset{\approx}{npc}\right)_{241}=\left(0,0,0,0,0\right)$	$\left(\overset{\approx}{npc}\right)_{242}=\left(0,0,0,0,0\right)$
$\left(\overset{\approx}{npc}\right)_{251}=\left(0,0,0,0,0\right)$	$\left(\overset{\approx}{npc}\right)_{252}=\left(0,0,0,0,0\right)$
$\left(\overset{\approx}{pt}\right)_{111}=\left(86,87,88,0.5,0.5\right)$	$\left(\overset{\approx}{pt}\right)_{112}=\left(86,87,88,0.5,0.5\right)$
$\left(\overset{\approx}{pt}\right)_{121}=\left(200,201,202,0.5,0.5\right)$	$\left(\overset{\approx}{pt}\right)_{122}=\left(200,201,202,0.5,0.5\right)$
$\left(\overset{\approx}{pt}\right)_{131}=\left(120,121,122,0.5,0.5\right)$	$\left(\overset{\approx}{pt}\right)_{132}=\left(120,121,122,0.5,0.5\right)$
$\left(\overset{\approx}{pt}\right)_{141}=\left(174,175,176,0.5,0.5\right)$	$\left(\overset{\approx}{pt}\right)_{142}=\left(174,175,176,0.5,0.5\right)$
$\left(\overset{\approx}{pt}\right)_{151}=\left(140,141,142,0.5,0.5\right)$	$\left(\overset{\approx}{pt}\right)_{152}=\left(140,141,142,0.5,0.5\right)$
$\left(\overset{\approx}{pt}\right)_{211}=\left(140,141,142,0.5,0.5\right)$	$\left(\overset{\approx}{pt}\right)_{212}=\left(140,141,142,0.5,0.5\right)$
$\left(\overset{\approx}{pt}\right)_{221}=\left(324,325,326,0.5,0.5\right)$	$\left(\overset{\approx}{pt}\right)_{222}=\left(324,325,326,0.5,0.5\right)$
$\left(\overset{\approx}{pt}\right)_{231}=\left(212,213,214,0.5,0.5\right)$	$\left(\overset{\approx}{pt}\right)_{232}=\left(212,213,214,0.5,0.5\right)$
$\left(\overset{\approx}{pt}\right)_{241}=\left(130,131,132,0.5,0.5\right)$	$\left(\overset{\approx}{pt}\right)_{242}=\left(130,131,132,0.5,0.5\right)$

$\left(\overset{\approx}{pt}\right)_{251} = (14,15,16,0.5,0.5)$	$\left(\overset{\approx}{pt}\right)_{252} = (14,15,16,0.5,0.5)$
$\left(\overset{\approx}{npt}\right)_{111} = (43,44,45,0.5,0.5)$	$\left(\overset{\approx}{npt}\right)_{112} = (43,44,45,0.5,0.5)$
$\left(\overset{\approx}{npt}\right)_{121} = (100,101,102,0.5,0.5)$	$\left(\overset{\approx}{npt}\right)_{122} = (100,101,102,0.5,0.5)$
$\left(\overset{\approx}{npt}\right)_{131} = (60,61,62,0.5,0.5)$	$\left(\overset{\approx}{npt}\right)_{132} = (60,61,62,0.5,0.5)$
$\left(\overset{\approx}{npt}\right)_{141} = (87,88,89,0.5,0.5)$	$\left(\overset{\approx}{npt}\right)_{142} = (87,88,89,0.5,0.5)$
$\left(\overset{\approx}{npt}\right)_{151} = (70,71,72,0.5,0.5)$	$\left(\overset{\approx}{npt}\right)_{152} = (70,71,72,0.5,0.5)$
$\left(\overset{\approx}{npt}\right)_{211} = (70,71,72,0.5,0.5)$	$\left(\overset{\approx}{npt}\right)_{212} = (70,71,72,0.5,0.5)$
$\left(\overset{\approx}{npt}\right)_{221} = (162,163,164,0.5,0.5)$	$\left(\overset{\approx}{npt}\right)_{222} = (162,163,164,0.5,0.5)$
$\left(\overset{\approx}{npt}\right)_{231} = (106,163,164,0.5,0.5)$	$\left(\overset{\approx}{npt}\right)_{232} = (106,163,164,0.5,0.5)$
$\left(\overset{\approx}{npt}\right)_{241} = (65,66,67,0.5,0.5)$	$\left(\overset{\approx}{npt}\right)_{242} = (65,66,67,0.5,0.5)$
$\left(\overset{\approx}{npt}\right)_{251} = (7,8,9,0.5,0.5)$	$\left(\overset{\approx}{npt}\right)_{252} = (7,8,9,0.5,0.5)$

Using the input value in Table-1.1, the objective functions together with crisp constraints are solved separately using GRG technique (lingo 18.0).

The feasible expected values is obtained under Critical value reduction and the expected values f_i^0 and $f_i^1 = (i = 1,2,3,4)$ are listed below:

Peak Time		Non-peak Time	
$f_1^0 = 1869546$	$f_1^1 = 1747957$	$f_3^0 = 1988174$	$f_3^1 = 1853344$
$f_2^0 = 24660$	$f_1^2 = 24124$	$f_4^0 = 23366$	$f_4^1 = 23093$

The membership functions for the objective functions f_1, f_2, f_3 and f_4 are formulated as follows

$$\mu_1\left(f_1(x)\right) = \begin{cases} 1, & for\ f_1(x) > 1869546 \\ \dfrac{f_1(x) - 1747957}{1869546 - 1747957}, & for\ 1747957 \le f_1(x) \le 1869546 \\ 0, & for\ f_1(x) < 1747957 \end{cases} \tag{15}$$

$$\mu_2\left(f_2(x)\right)=\begin{cases}1, & for\ f_2(x)>24660\\ \dfrac{24660-f_2(x)}{24660-24124} & for\ 1975806\le f_2(x)\le 2208444\\ 0, & for\ f_2(x)<24124\end{cases} \qquad (16)$$

Similarly, the membership functions can be formulated for the objective functions f_3 and f_4

The following model is formulated using fuzzy goal programming (FGPT) technique and the critical value is give as follows.

PEAK TIME:

Maximize $\left[d_1^- + d_2^-\right]$

subject to the constraints:

$$\begin{cases}\dfrac{f_1-1747957}{1869546-1747957}+d_1^+ - d_1^- = \mu_1\\ \dfrac{f_2-24660}{24660-24124}+d_2^+ - d_2^- = \mu_2\\ d_i^+.d_i^- = 0, d_i^+.d_i^- \ge 0, i=1,2\\ d_i^+ \ge 0, d_i^- \ge 0, i=1,2\end{cases} \qquad (17)$$

Similarly the above model can be formulated for non-peak time. The above single objective function is solved using LINGO-18.0 and the optimum results are reported in the following table.

Table 1.2: Expected critical values via FGPT (peak time and Non-peak time)

μ_1	μ_2	μ_3	μ_4	f_1	f_2	f_3	f_4
0.2	0.8	0.2	0.8	1845229	24321	1857231	24321
0.3	0.7	0.3	0.7	1853070	24117	1858476	24117
0.4	0.6	0.4	0.6	1848127	24432	1865380	24432
0.5	0.5	0.5	0.5	1812803	24562	1872753	24562

The following model is formulated using weighted fuzzy goal programming technique (WFGPT) (for critical value)

PEAK TIME:

Maximize $\sum_{i=1}^{2}\left[w_i\left(d_i^+ + d_i^-\right)\right]$

$$\text{subject to the constraints:}\begin{cases}\dfrac{f_1-1747957}{1869546-1747957}+d_1^+-d_1^-=w_1\\ \dfrac{f_2-24660}{24660-24124}+d_2^+-d_2^-=w_2\\ d_i^+.d_i^-=0, d_i^+.d_i^-\geq 0, i=1,2\\ d_i^+\geq 0, d_i^-\geq 0, i=1,2\end{cases}\tag{18}$$

Similarly the above model can be formulated for non-peak time. The above single objective function is solved using LINGO-18.0 and the optimum results are reported in the following table.

Table 1.3: Expected critical values via WFGPT (peak time and non-peak time)

w_1	w_2	w_3	w_4	f_1	f_2	f_3	f_4
0.2	0.8	0.2	0.8	1760115	25088	1866827	23584
0.3	0.7	0.3	0.7	1784433	25035	1893793	23557
0.4	0.6	0.4	0.6	1796592	24981	1907276	23529
0.5	0.5	0.5	0.5	1808751	24928	1920759	23502

From the above tables it is observed that the profit in transportation increases with respect to increase in credibility measure and the time required for transportation decreases with increase in credibility measures.

Table1.4: Percentage of profit increase in non- peak time using FGPT & WFGPT

μ_3	μ_4	percentage of profit via FGPT	W_3	W_4	Percentage of profit via WFGPT
0.2	0.8	0.32	0.2	0.8	2
0.3	0.7	0.14	0.3	0.7	2.99
0.4	0.6	0.46	0.4	0.6	2.98
0.5	0.5	1.6	0.5	0.5	3

Table 1.5: Percentage of time reduction in non-peak time using FGPT & WFGPT

μ_3	μ_4	percentage of time reduction via FGPT	W_3	W_4	percentage of time reduction via WFGPT
0.2	0.8	2.12	0.2	0.8	3
0.3	0.7	1.75	0.3	0.7	3.04
0.4	0.6	2.46	0.4	0.6	2.99
0.5	0.5	2.78	0.5	0.5	2.94

8. Conclusion

This paper mainly investigated multi objective profit transportation problem with congestion for multi items under type-2 fuzzy environment. Using the numerical experiment for the proposed model, it is observed that the profit is increased in non-peak time than the profit obtained in peak time. The profit increase in percentage is shown in the table 1.4. Also, the transportation time is minimized in non-peak time than peak time.

The percentage of reduction in time is given in the table 1.5. The present paper can be extended to different types of transportation problems including price discounts, breakable items and damageable items and also for transshipment problem in complex environment.

REFERENCES

1. Animal Husbandry, Dairying and Fisheries Department, Government of Tamil Nadu, 41 (2018) 3297-3205.
2. R.E.Bellman and l.A.Zadeh, Decision making in a fuzzy environment, Management Sciences, 17 (1970) B141-B164.
3. A.Charnes and W.Cooper, Management Models and Industrial Applications of Linear Programming, Wiley, New York, vol 1, (1961).
4. S.Chanas and W.Kolodziejckzy, A Fuzzy approach to the Transportation Problem, Fuzzy Sets Syst., 13 (1984) 211-221.
5. Y.Chen, The generalized expectations of the reductions for type-2 fuzzy variable, Machine Learning and Cybernetics (ICMLC), IEEE Vol.1, (2011),.
6. S.Chanas and D.Kuchata, A concept of the optimal solution of the transportation problem with fuzzy cost coefficients, Fuzzy Sets System, 82 (1996) 299-305.
7. D.K.Jana, Comparative study on credibility measures of type-2 and type-1 fuzzy variables and their application to a multi-objective profit transportation problem via Goal Programming, Intern. J.Transportation Science and Technology, (2017) 110-126.
8. D.Dubois and H.Prode, Possibility Theory, An Approach to Computerized processing of uncertainty, Plenum Press, New York, (1988).
9. A.Dutta and D.K.Jana, Expectations of reductions for type-2 trapezoidal fuzzy variables and its application to a multi-objective solid transportation problem via goal programming technique, J. Uncertainty Anal. Appl., 51 (2017) 3-28.
10. J.Mendel and I.Bob John, Type-2 Fuzzy Sets Made Simple, IEEE Transactions on Fuzzy Systems, 10(2) (2002) 134-145.
11. F.L.Hitchcock, The distribution of a product from several sources to numerous localities, Journal of Mathematics and Physics, 20 (1941) 224-230.
12. G.Klir, On Fuzzy set interpretation of possibility theory, Fuzzy Sets and Systems, 108 (2000) 535-550.
13. N.N.Karnik and J.M.Mendel, Operations on Type-2 fuzzy logic systems, IEEE Trans. Fuzzy System, 8(5) (1984) 211-221.
14. Z.Q.Liu and Y.K.Liu, Type-2 fuzzy variable and their arithmetic, Soft Computing, 14 (2010) 729-747.
15. M.OhEigeartaigh, A fuzzy transportation algorithm, Fuzzy Sets and Systems, 8 (1982) 235-243.
16. R.Qin, Y.Liu and Z.Liu, Methods of critical value reduction for type-2 fuzzy variables and their applications, J. Maths. and Appl., 235 (2011) 1454-1481.
17. O.M.Saad and S.A.Abbas, A parametric study on transportation problem under fuzzy environment, J. Fuzzy Math, 11 (2003) 115-124.
18. Y.Yafeng, S.Phong Lawphongpanich, Internalizing emission externality on road networks, Transportation Research Part D, 11 (2006) 292-301.

19. L.A.Zadeh, The concept of a linguistic variable and its application in approximate reasoning, Inf., Sci., 8 (1975) 199-249.
20. L.A.Zadeh, Fuzzy Sets as a basis for a theory of possibility, Fuzzy Sets and Systems, 1 (1978) 3-28.
21. L.A.Zadeh, Fuzzy Sets, Information and Control, 8 (1965) 338-353.
22. H.J.Zimmermann, Fuzzy programming and linear programming with several objective functions, Fuzzy Sets and System, 1 (1978) 45-46.