Journal of Physical Sciences, Vol.24, 2019, 53-61
ISSN: 2350-0352 (print), www.vidyasagar.ac.in/publication/journal
Published on 6 June 2019

Solving Critical Path in Project Scheduling by using TOPSIS Ranking of Generalized Interval Valued Octagonal Fuzzy Numbers

D.Stephen Dinagar[1] and N.Rameshan[2]

[1]*PG and Research Department of Mathematics*, *TBML College*
Porayar – 609307, India. Email: dsdina@rediffmail.com

[2]*Department of Mathematics, SRM Institute of Science and Technology*
Vadapalani, Chennai – 600026, India
Email: nrameshan14@gmail.com

Received 29 December 2018; accepted 31 January 2019

ABSTRACT

This paper presents TOPSIS ranking method using Octagonal fuzzy number applied in a critical path in fuzzy environment. Using the above said notion, we convert fuzzy critical activities into crisp critical activities. New algebraic arithmetic of Generalized Interval valued octagonal fuzzy numbers (GIOCFNs) is investigated. A suitable numerical example is discussed to understand the method.

Keywords: Octagonal Fuzzy numbers, Generalized Interval valued octagonal fuzzy number, Critical path method, Fuzzy project network, Fuzzy ranking method, Interval valued fuzzy numbers.

Mathematical Subject Classification (2010): 94D05

1. Introduction

In this work, by applying the procedure of the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), it is proposed a method based on GIOCFNs for solving critical path in project network.

In earlier, Parida and Sahoo [1] developed the procedure Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) is multiple criteria decision analysis method (MCDA), which is chosen alternatives, is nearest to the positive ideal solution and farthest from negative ideal solution. A common generalization of TOPSIS is extensions of TOPSIS under fuzzy environment were identified to overcome the difficulties in exact data of real situation. Here, GIOCFNs were used to the interval valued fuzzy environment to improve the new approach based on actual procedure of TOPSIS.

Chen [2] extended the TOPSIS procedure to fuzzy group decision making situations. Dinagar and Abirami [3, 4] were found the critical path by TOPSIS method. Stephen Dinagar and Thiripurasundari [5] discussed critical path problems using TOPSIS. From Chen [6], Chen and Chen [7] and Wei and Chen [8] arithmetic operation between A and B were reviewed in the Definition 2.7. Neelima et al. [9] used fuzzy TOPSIS method in objective and systematic evaluation of alternatives on multiple criteria.

2. Preliminaries

In this section we define some basic definitions which will be applied in this paper.

Definition 2.1. Let X be a set. A fuzzy set $\tilde{A}$ on X is defined to be a function $\mu_{\tilde{A}}:X\rightarrow[0,1]$ is a mapping called the membership value of $x\in X$ in a fuzzy set $\tilde{A}$.

Definition 2.2. The fuzzy number $\tilde{A}$ is fuzzy set if membership function satisfies

i) A fuzzy set of the universe of discourse X is convex

ii) $\tilde{A}$ is normal if $\exists\, x_i\in X$, $\mu_{\tilde{A}}(x_i)=1$

iii) $\mu_{\tilde{A}}(x)$ is piecewise continuous

Definition 2.3. A fuzzy number $\tilde{A}$ is a normal octagonal fuzzy number denoted by $(a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8)$ which real numbers and its membership function are $\mu_{\tilde{A}}(x)$ is given by

$$\mu_{\tilde{A}}(x)=\begin{cases} 0, & x\le a_1 \\ k\left(\dfrac{x-a_1}{a_2-a_1}\right), & a_1\le x\le a_2 \\ k, & a_2\le x\le a_3 \\ k+(1-k)\left(\dfrac{x-a_3}{a_4-a_3}\right), & a_3\le x\le a_4 \\ 1, & a_4\le x\le a_5 \\ k, & a_6\le x\le a_7 \\ k\left(\dfrac{a_8-x}{a_8-a_7}\right), & a_7\le x\le a_8 \\ 0, & x\ge a_8 \end{cases}$$

where $0<k<1$

Result: If k=0 and k=1 then OCFN reduces to trapezoidal fuzzy number (a_3,a_4,a_5,a_6) and trapezoidal fuzzy number (a_1,a_4,a_5,a_8) respectively.

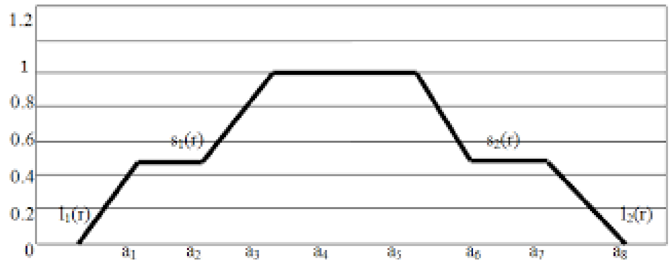


Figure 2.1: Octagonal fuzzy number for k=0.5

Remark: A fuzzy number $\widetilde{A}$ is positive $(\widetilde{A}>0)$ if $\mu_{\widetilde{A}}(x)=0\ \forall\ x\leq 0$ and $\widetilde{A}$ is negative $(\widetilde{A}<0)$, if $\mu_{\widetilde{A}}(x)=0\ \forall\ x\geq 0$

Definition 2.4. Let X be nonempty finite set. An IVFS $\tilde{A}$ is defined by $\widetilde{A}=\left\{x,\frac{\mu_A(x)}{x}\in X\right\}$, where $\mu_{\widetilde{A}}: X\to Int(|0,1|)$ defines the degree of membership of x to $\widetilde{A}$, $\ni$, $x\to\mu_{\widetilde{A}}(x)=\left[\mu^-_{\widetilde{A}}(x),\mu^+_{\widetilde{A}}(x)\right]$.

Definition 2.5. Let X∈IVFS(X). If convex set A(x) in closed and bounded interval then A is said to "A generalized interval valued fuzzy number (IVFN) on the universe of discourse X." Let $\widetilde{A}(x)=\left[\widetilde{A}^L(x),\widetilde{A}^U(x)\right]$, where $0\leq\widetilde{A}^L(x)\leq\widetilde{A}^U(x)\leq 1$, $x\in X$, $\widetilde{A}^L:X\to[0,1]$ and $\widetilde{A}^U:X\to[0,1]$.

3. Generalized interval valued fuzzy octagonal fuzzy number
In this section, we discussed with notion of GIOCFN and their arithmetic operations.

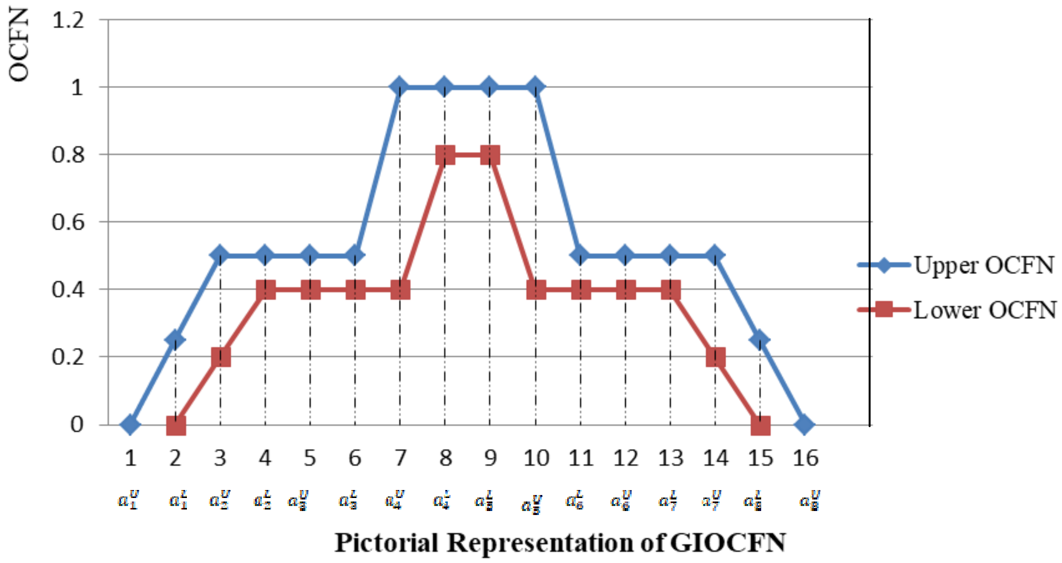
Definition 3.1. Let $\widetilde{A}^L\&\widetilde{A}^U$ be two generalized octagonal fuzzy numbers and $h^L_{\widetilde{A}}\&h^U_{\widetilde{A}}$ denote heights of $\widetilde{A}^L\&\widetilde{A}^U$ respectively.
Let $a^L_1,a^L_2,a^L_3,a^L_4,a^L_5,a^L_6,a^L_7,a^L_8$, $a^U_1,a^U_2,a^U_3,a^U_4,a^U_5,a^U_6,a^U_7,a^U_8$ be real values.
A GIOCFN $\tilde{A}$ defined as

$$\widetilde{A}=\left[\widetilde{A}^L,\widetilde{A}^U\right]=\left[\left(a^L_1,a^L_2,a^L_3,a^L_4,a^L_5,a^L_6,a^L_7,a^L_8;h^L_A\right),\left(a^U_1,a^U_2,a^U_3,a^U_4,a^U_5,a^U_6,a^U_7,a^U_8;h^U_A\right)\right]$$

where $a^L_1\leq a^L_2\leq a^L_3\leq a^L_4\leq a^L_5\leq a^L_6\leq a^L_7\leq a^L_8$, $a^U_1\leq a^U_2\leq a^U_3\leq a^U_4\leq a^U_5\leq a^U_6\leq a^U_7\leq a^U_8$, $0\leq h^L_{\widetilde{A}}\leq h^U_{\widetilde{A}}\leq 1$, $a^U_1\leq a^L_1$ and $a^L_8\leq$

Definition 3.2. The membership function $\mu_A: x\to[0,1]$ of GIOCFN A is defined as

$$\mu_A(x)=\begin{cases}\dfrac{x-a_1^U}{a_2^U-a_1^U}, a_1^U\le x\le a_2^U\\ \dfrac{x-a_1^L}{a_2^L-a_1^L}, a_1^L\le x\le a_2^L\\ \dfrac{x-a_2^U}{a_3^U-a_2^U}, a_2^U\le x\le a_3^U\\ \dfrac{x-a_2^L}{a_3^L-a_2^L}, a_2^L\le x\le a_3^L\\ \dfrac{x-a_3^U}{a_4^U-a_3^U}, a_3^U\le x\le a_4^U\\ \dfrac{x-a_3^L}{a_4^L-a_3^L}, a_3^L\le x\le a_4^L\\ \dfrac{x-a_4^L}{a_5^L-a_4^L}, a_4^L\le x\le a_5^L\\ 1, \quad a_4^U\le x\le a_5^U\\ \dfrac{a_5^L-x}{a_6^L-a_5^L}, a_5^L\le x\le a_6^L\\ \dfrac{a_5^U-x}{a_6^U-a_5^U}, a_5^U\le x\le a_6^U\\ \dfrac{a_6^L-x}{a_7^L-a_6^L}, a_6^L\le x\le a_7^L\\ \dfrac{a_6^U-x}{a_7^U-a_6^U}, a_6^U\le x\le a_7^U\\ \dfrac{a_7^L-x}{a_8^L-a_7^L}, a_7^L\le x\le a_8^L\\ \dfrac{a_7^U-x}{a_8^U-a_7^U}, a_7^U\le x\le a_8^U\\ 0, \quad Otherwise\end{cases}$$



Remark:

i) GIOCFN contains two generalized octagonal fuzzy numbers the lower octagonal fuzzy number $\widetilde{A}^{L}$ & upper octagonal fuzzy number $\widetilde{A}^{U}$

ii) GIOCFN converts normal interval valued octagonal fuzzy number, when $h^{L}_{\widetilde{A}}=h^{U}_{\widetilde{A}}=1$

iii) A said to be generalized octagonal fuzzy number, if $\widetilde{A}^{L}=\widetilde{A}^{U}$

Definition 3.3. (Arithmetic operations of GIOCFN)

If $\widetilde{A}=\left[\widetilde{A}^{L},\widetilde{A}^{U}\right]=\left[\left(a^{L}_{1},a^{L}_{2},a^{L}_{3},a^{L}_{4},a^{L}_{5},a^{L}_{6},a^{L}_{7},a^{L}_{8};h^{L}_{A}\right),\left(a^{U}_{1},a^{U}_{2},a^{U}_{3},a^{U}_{4},a^{U}_{5},a^{U}_{6},a^{U}_{7},a^{U}_{8};h^{U}_{A}\right)\right]$ and $\widetilde{B}=\left[\widetilde{B}^{L},\widetilde{B}^{U}\right]=\left[\left(b^{L}_{1},b^{L}_{2},b^{L}_{3},b^{L}_{4},b^{L}_{5},b^{L}_{6},b^{L}_{7},b^{L}_{8};h^{L}_{B}\right),\left(b^{U}_{1},b^{U}_{2},b^{U}_{3},b^{U}_{4},b^{U}_{5},b^{U}_{6},b^{U}_{7},b^{U}_{8};h^{U}_{\widetilde{B}}\right)\right]$ then,

i) Addition of GIOCFN

$$\widetilde{A}+\widetilde{B}=[\left(a^{L}_{1}+b^{L}_{1},a^{L}_{2}+b^{L}_{2},a^{L}_{3}+b^{L}_{3},a^{L}_{4}+b^{L}_{4},a^{L}_{5}+b^{L}_{5},a^{L}_{6}+b^{L}_{6},a^{L}_{7}+b^{L}_{7},a^{L}_{8}+b^{L}_{8};\min\left(h^{L}_{\widetilde{A}},h^{L}_{\widetilde{B}}\right)\right)$$
$$(a^{U}_{1}+b^{U}_{1},\ a^{U}_{2}+b^{U}_{2},a^{U}_{3}+b^{U}_{3},a^{U}_{4}+b^{U}_{4},a^{U}_{5}+b^{U}_{5},a^{U}_{6}+b^{U}_{6},a^{U}_{7}+b^{U}_{7},a^{U}_{8}+b^{U}_{8};\ \min\ (h^{U}_{A},\ h^{U}_{B}))]$$

ii) Subtraction of GIOCFN

$$\widetilde{A}-\widetilde{B}=[((a^{L}_{1}-b^{L}_{8},a^{L}_{2}-b^{L}_{7},a^{L}_{3}-b^{L}_{6},a^{L}_{4}-b^{L}_{5},a^{L}_{5}-b^{L}_{4},a^{L}_{6}-b^{L}_{3},a^{L}_{7}-b^{L}_{2},a^{L}_{8}-b^{L}_{1};\min\left(h^{L}_{\widetilde{A}},h^{L}_{\widetilde{B}}\right)$$
$$(a^{U}_{1}-b^{U}_{8},\ a^{U}_{2}-b^{U}_{7},a^{U}_{3}-b^{U}_{6},a^{U}_{4}-b^{U}_{5},a^{U}_{5}-b^{U}_{4},a^{U}_{6}-b^{U}_{3},a^{U}_{7}-b^{U}_{2},a^{U}_{8}-b^{U}_{1};\ \min\ (h^{U}_{A},\ h^{U}_{\widetilde{B}}))]$$

iii) Multiplication of GIOCFN

$$\widetilde{A}\times\widetilde{B}=[\left(a^{L}_{1}\times b^{L}_{1},a^{L}_{2}\times b^{L}_{2},a^{L}_{3}\times b^{L}_{3},a^{L}_{4}\times b^{L}_{4},a^{L}_{5}\times b^{L}_{5},a^{L}_{6}\times b^{L}_{6},a^{L}_{7}\times b^{L}_{7},a^{L}_{8}\times b^{L}_{8};\min\left(h^{L}_{\widetilde{A}},h^{L}_{\widetilde{B}}\right)\right)$$
$$(a^{U}_{1}\times b^{U}_{1},\ a^{U}_{2}\times b^{U}_{2},a^{U}_{3}\times b^{U}_{3},a^{U}_{4}\times b^{U}_{4},a^{U}_{5}\times b^{U}_{5},a^{U}_{6}\times b^{U}_{6},a^{U}_{7}\times b^{U}_{7},a^{U}_{8}\times b^{U}_{8};\ \min(h^{U}_{A},\ h^{U}_{B}))]$$

iv) Division of GIOCFN

$$\widetilde{A}/\widetilde{B}=[\left(a^{L}_{1}/b^{L}_{8},a^{L}_{2}/b^{L}_{7},a^{L}_{3}/b^{L}_{6},a^{L}_{4}/b^{L}_{5},a^{L}_{5}/b^{L}_{4},a^{L}_{6}/b^{L}_{3},a^{L}_{7}/b^{L}_{2},a^{L}_{8}/b^{L}_{1};\min\left(h^{L}_{\widetilde{A}},h^{L}_{\widetilde{B}}\right)\right)$$
$$(a^{U}_{1}/b^{U}_{8},\ a^{U}_{2}/b^{U}_{7},a^{U}_{3}/b^{U}_{6},a^{U}_{4}/b^{U}_{5},a^{U}_{5}/b^{U}_{4},a^{U}_{6}/b^{U}_{3},a^{U}_{7}/b^{U}_{2},a^{U}_{8}/b^{U}_{1};\ \min\ (h^{U}_{A},\ h^{U}_{B}))]$$

Definition 3.4. The Distance between any two IVFNs $\widetilde{A}=\left[\left(a^{L}_{1},a^{L}_{2},a^{L}_{3},a^{L}_{4},a^{L}_{5},a^{L}_{6},a^{L}_{7},a^{L}_{8},\right),\left(a^{U}_{1},a^{U}_{2},a^{U}_{3},a^{U}_{4},a^{U}_{5},a^{U}_{6},a^{U}_{7},a^{U}_{8}\right)\right]$ and $\widetilde{B}=\left[\left(b^{L}_{1},b^{L}_{2},b^{L}_{3},b^{L}_{4},b^{L}_{5},b^{L}_{6},b^{L}_{7},b^{L}_{8},\right),\left(b^{U}_{1},b^{U}_{2},b^{U}_{3},b^{U}_{4},b^{U}_{5},b^{U}_{6},b^{U}_{7},b^{U}_{8}\right)\right]$ be defined as

$$D\left(\widetilde{A},\widetilde{B}\right)=\frac{1}{8}\max\begin{Bmatrix}\left|\left(a^{L}_{1}+a^{U}_{1}\right)-\left(b^{L}_{1}+b^{U}_{1}\right)\right|+\left|\left(a^{L}_{8}+a^{U}_{8}\right)-\left(b^{L}_{8}+b^{U}_{8}\right)\right|,\\ \left|\left(a^{L}_{2}+a^{U}_{2}\right)-\left(b^{L}_{2}+b^{U}_{2}\right)\right|+\left|\left(a^{L}_{7}+a^{U}_{7}\right)-\left(b^{L}_{7}+b^{U}_{7}\right)\right|,\\ \left|\left(a^{L}_{3}+a^{U}_{3}\right)-\left(b^{L}_{3}+b^{U}_{3}\right)\right|+\left|\left(a^{L}_{6}+a^{U}_{6}\right)-\left(b^{L}_{6}+b^{U}_{6}\right)\right|,\\ \left|\left(a^{L}_{4}+a^{U}_{4}\right)-\left(b^{L}_{4}+b^{U}_{4}\right)\right|+\left|\left(a^{L}_{5}+a^{U}_{5}\right)-\left(b^{L}_{5}+b^{U}_{5}\right)\right|\end{Bmatrix}$$

For $\widetilde{B}=\tilde{1}$, $$D(\widetilde{A},\tilde{1})=\frac{1}{8}\max\left\{\begin{array}{l}\left|(a_1^L+a_1^U)-(1+1)\right|+\left|(a_8^L+a_8^U)-(1+1)\right|,\\ \left|(a_2^L+a_2^U)-(1+1)\right|+\left|(a_7^L+a_7^U)-(1+1)\right|,\\ \left|(a_3^L+a_3^U)-(1+1)\right|+\left|(a_6^L+a_6^U)-(1+1)\right|,\\ \left|(a_4^L+a_4^U)-(1+1)\right|+\left|(a_5^L+a_5^U)-(1+1)\right|\end{array}\right\}$$

For $\widetilde{B}=\tilde{0}$, $$D(\widetilde{A},\tilde{0})=\frac{1}{8}\max\left\{\begin{array}{l}\left|(a_1^L+a_1^U)-(0+0)\right|+\left|(a_8^L+a_8^U)-(0+0)\right|,\\ \left|(a_2^L+a_2^U)-(0+0)\right|+\left|(a_7^L+a_7^U)-(0+0)\right|,\\ \left|(a_3^L+a_3^U)-(0+0)\right|+\left|(a_6^L+a_6^U)-(0+0)\right|,\\ \left|(a_4^L+a_4^U)-(0+0)\right|+\left|(a_5^L+a_5^U)-(0+0)\right|\end{array}\right\}$$

Remark: The distance from A to $\tilde{1}_1$

For $0\le h_A^L\le h_A^U\le 1$

$$d(\widetilde{A},\tilde{1}_1)=\frac{1}{16}\left(a_1^L+a_2^L+a_3^L+a_4^L+a_5^L+a_6^L+a_7^L+a_8^L+a_1^U+a_2^U+10(a_3^U+a_4^U+a_5^U+a_6^U)+a_7^U+a_8^U)\frac{h_A^L}{h_A^U}-32\right)$$

For $0\le h_A^L=h_A^U\le 1$

$$d(\widetilde{A},\tilde{1}_1)=\frac{1}{16}\left(a_1^L+a_2^L+a_3^L+a_4^L+a_5^L+a_6^L+a_7^L+a_8^L+a_1^U+a_2^U+10(a_3^U+a_4^U+a_5^U+a_6^U)+a_7^U+a_8^U-32\right)$$

4. TOPSIS algorithm

Technique for Order Performance by Similarity to Ideal Solution (TOPSIS) procedure can be used to find multiple alternatives of selected criteria. Here, alternative is smallest distance to Fuzzy Positive Ideal solution (FPIS) and longest distance to Fuzzy negative ideal solution (FNIS).

4.1. Procedure

Let us consider, there are k members in the decision group. In k^{th} decision maker, the fuzzy rating and importance weight w.r.t. j^{th} alternative on j^{th} criterion.

Step 1:

For $\widetilde{A}=[(a_1^L,a_2^L,a_3^L,a_4^L,a_5^L,a_6^L,a_7^L,a_8^L,),(a_1^U,a_2^U,a_3^U,a_4^U,a_5^U,a_6^U,a_7^U,a_8^U)]$ be Interval Valued Fuzzy number (IVFN), construct the fuzzy decision matrix $\widetilde{D}^i$

Step 2:

Normalize each fuzzy number $\tilde{A}$ such that

$\widetilde{A}_{\widetilde{N}^i}=\left[\left(\frac{a_1^L}{k},\frac{a_2^L}{k},\frac{a_3^L}{k},\frac{a_4^L}{k},\frac{a_5^L}{k},\frac{a_6^L}{k},\frac{a_7^L}{k},\frac{a_8^L}{k}\right),\left(\frac{a_1^U}{k},\frac{a_2^U}{k},\frac{a_3^U}{k},\frac{a_4^U}{k},\frac{a_5^U}{k},\frac{a_6^U}{k},\frac{a_7^U}{k},\frac{a_8^U}{k}\right)\right]$, where k refers the paired minimum ranking of activities. Let the normalized decision matrix be $\widetilde{N}^i$.

Step 3:

Find the weighted normalized fuzzy decision matrix.

Let us consider $\left[\left(a_1^L,a_2^L,a_3^L,a_4^L,a_5^L,a_6^L,a_7^L,a_8^L\right),\left(a_1^U,a_2^U,a_3^U,a_4^U,a_5^U,a_6^U,a_7^U,a_8^U\right)\right]$be the vertices of a octagonal is given by

$$\begin{bmatrix}\left((a_1^L,0),(a_2^L,0.5),(a_3^L,0.5),(a_4^L,1),(a_5^L,1),(a_6^L,0.5),(a_7^L,0.5),(a_8^L,0)\right),\\ \left((a_1^U,0),(a_2^U,0.5),(a_3^U,0.5),(a_4^U,1),(a_5^U,1),(a_6^U,0.5),(a_7^U,0.5),(a_8^U,0)\right)\end{bmatrix}$$

and find the average area of the two octagonal. Divide the area of each IVFN by maximum area among all the areas to fix its weight. Associate each weight to form the weighted matrix $\widetilde{W}^i$of $\widetilde{D}^i$.

Step 4:

Calculate the weighted normalized decision matrix. The weighted normalized value $\widetilde{V}^i = \widetilde{N}^i \times R(\widetilde{W}^i)$.

Step 5:

Find FPIS $\tilde{1}$=[(1,1,1,1,1,1,1,1),(1,1,1,1,1,1,1,1)] ,

FNIS $\tilde{0}$= [(0,0,0,0,0,0,0,0),(0,0,0,0,0,0,0,0)]

Step 6:

Find distance, from alternative to FPIS $d^{+i} = \sum D(\tilde{A},\tilde{1})$, and from alternative to FNIS$d^{-i} = \sum D(\tilde{A},\tilde{0})$

Step 7:

Compute the closeness coefficient $CC_i = \frac{\tilde{d}^-}{\tilde{d}^+ + \tilde{d}^-}$for each alternative

Step 8:

Find alternative with the highest closeness coefficient gives the best alternative.

5. Numerical illustration

Let us consider the project network with 7 nodes whose activities are in hours.

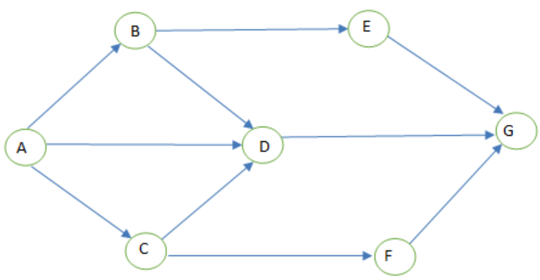


Figure 2.2: Project network

Table 1: Fuzzy activity time

Activity	**A-B**	**A-C**	**A-D**
Fuzzy Duration (Hours)	Approx. b/w 10-15 hours [(10,12,11,13,15,12,11,13) (10,11,12,13,10,14,15,13)]	Approx. b/w 14-20 hours [(14,16,15,17,20,14,19,15) (14,15,17,16,20,15,14,15)]	Approx. b/w 15-23 hours [(15,20,16,23,15,21,18,22) (16,15,22,15,20,20,23,18)]
Activity	**B-D**	**B-E**	**C-F**
Fuzzy Duration (Hours)	Approx. b/w 8-15 hours [(8,9,10,11,15,12,13,10) (9,10,12,11,14,13,10,15)]	Approx. b/w 5-11 hours [(5,8,7,6,9,6,10,11) (6,5,8,10,11,7,6,7)]	Approx. b/w 5-13 hours [(5,7,6,11,13,10,7,12) (6,7,13,11,12,10,7,6)]
Activity	**D-F**	**D-G**	**E-G**
Fuzzy Duration (Hours)	Approx. b/w 13-18 hours [(13,15,14,15,16,14,15,13) (17,13,15,13,15,14,16,18)]	Approx. b/w 20-26 hours [(20,22,23,26,25,24,21,22) (20,23,25,22,20,21,26,22)]	Approx. b/w 15-19 hours [(15,16,17,18,15,17,16,19) (16,17,18,15,16,17,19,18)]
Activity	**F-G**	--	--
Fuzzy Duration (Hours)	Approx. b/w 13-20 hours [(13,18,17,18,20,14,20,19) (15,13,18,16,19,17,14,20)]	--	--

Calculation:

	A-B	**A-C**	**A-D**	**B-D**	**B-E**	**C-F**	**D-F**	**D-G**	**E-G**	**F-G**
$\mathbf{d^{+i}}$	33.875	52.250	60.000	32.813	25.625	37.500	44.563	70.125	51.938	54.313
$\mathbf{d^{-i}}$	35.875	54.250	62.000	34.813	27.625	39.500	46.563	72.125	53.938	56.313
$\mathbf{CC^*}$	0.5143	0.5094	0.5082	0.5147	0.5187	0.5129	0.5098	0.5070	0.5094	0.5090

Path	**Closeness coefficient** CC^*
A-B-D-G	1.5360
A-B-D-F-G	2.0478
A-B-E-G	1.5420
A-C-F-G	1.5313
A-D-G	1.0152
A-D-F-G	1.5270

The critical path is A-D-G, which has minimum closeness coefficient.

6. Conclusion

In this paper, we represented the extension of TOPSIS approach for solving critical path in project network by using octagonal fuzzy numbers. This approach is very useful and effective in handling the critical problems with qualitative nature. This procedure can be also used in some other optimization methods in future.

REFERENCES

1. P.K.Parida and S.K.Sahoo, Multiple attribute decision making approach by TOPSIS technique, International Journal of Engineering Research & Technology, 11(2) (2013) 907-911.
2. C.T.Chen, Extensions of the TOPSIS for group decision-making under fuzzy environment, Fuzzy Sets and Systems, 114 (2000) 1-9.
3. D.Stephen Dinagar and D.Abirami, On L-R type interval valued fuzzy numbers in critical path analysis, International Journal of Fuzzy Mathematical Achieve, 6 (1) (2015) 77-83.
4. D.Stephen Dinagar and D.Abirami, A note on fuzzy critical path in project scheduling using topsis ranking method, International Journal of Applications of Fuzzy Sets and Artificial Intelligence, 6 (2016)5-15.
5. D.Stephen Dinagar and K.Thiripurasundari, The critical path for the project scheduling problem using TOPSIS method in fuzzy environment, ICRDPET, 6 (2013) 164-166.
6. Ting-Yu-Chen, Signed distanced-based TOPSIS method for multiple criteria decision analysis based on generalized interval-valued fuzzy numbers, International Journal of Information Technology & Decision Making, 10 (6) (2011) 1131-1159.
7. S.H.Wei and S.M.Chen, Fuzzy risk analysis based on interval-valued fuzzy numbers, Expert System with Applications, 36(2-1) (2009) 2285-2299.
8. S.J.Chen and S.M.Chen, Fuzzy risk analysis based on measures of similarity between interval- valued fuzzy numbers, Computers & Mathematics with Applications, 55(8) (2008) 1670-1685.
9. N.B.Kore, K.Ravi and S.B.Patil, A simplified description of fuzzy method for multi criteria decision making, International Research Journal of Engineering Technology, 4(5) (2017) 1-4.