Journal of Physical Sciences, Vol. 24, 2019, 39-44
ISSN: 2350-0352 (print), www.vidyasagar.ac.in/publication/journal
Published on 6 June 2019

A Study on Arithmetic Operations of Type-2 Triangular Mixed Fuzzy Numbers

***K. Thiripurasundari*[1], *K. Latha*[2] *and D. Valli*[3]**

PG and Research Department of Mathematics
Poompuhar College, Melaiyur, India
Email: mktps83@gmail.com

Received 8 December 2018; accepted 2 February 2019

ABSTRACT

The concept of a type-2 fuzzy set which is an extension of the concept of an ordinary fuzzy set was introduced by Zadeh. Type-2 fuzzy sets have grades of membership that are themselves fuzzy. Type-2 fuzzy sets possess a great expressive power and are conceptually quite appealing. This paper deals with addition, subtraction, multiplication and division of type-2 triangular mixed fuzzy numbers using alpha cut methods.

Keywords: type-2 fuzzy set, type-2 fuzzy number, type-2 triangular mixed fuzzy numbers

Mathematical Subject Classification (2010): 15B15

1. Introduction

Fuzzy methods allow the processing of imprecise and indecisive variables. Fuzzy methods come in two main flavors, type-1 and type–2. The concept of a type–2 fuzzy set, an extension of the concept of an ordinary fuzzy set, was introduced by Zadeh [8,9]. Type-1 fuzziness represents the first step toward the full processing of qualms and the type-2 fuzziness builds upon the strengths of the type–1 methods and overcomes some of the limitations. It is observed that the type-2 fuzzy paradigm is lesser known and lesser exploited area of fuzziness and also the type–2 fuzzy methods provide second order uncertainties allowing fuzzy systems to accurately deal with real world uncertainty. At present the full potential of type-2 methods has not been exploited by practitioners, probably because of computational expense of type-2 operations. In the current climate of ever faster, more dominant and more affordable hardware, the type-2 fuzzy methods present a stimulating opportunity to explore uncertainties in real world systems in ways that were not previously possible.

A type-2 fuzzy set is characterized by a membership function, ie, the membership value for each element of this set is a fuzzy set in [0,1]. The concept of a type-2 triangular fuzzy number was presented by Dinagar and Anbalagan [5]. The another form of type-2 triangular fuzzy number called type-2 triangular mixed fuzzy number was presented by Thiripurasundari [6,7]. Type-2 triangular fuzzy matrices and type-2 fuzzy linear equations was presented by Latha et al. [2,3]. This paper deals with various arithmetic operations using alpha cut method [1] for type-2 triangular mixed fuzzy numbers. In section -2, some basic definitions are given. In section – 3 deals with various arithmetic operations using alpha cut method for type – 2 triangular mixed fuzzy numbers. In section – 4, conclusion is included.

2. Preliminaries

2.1. Type-2 fuzzy set

A type-2 fuzzy set is a fuzzy set whose membership values are fuzzy sets on [0,1].

2.2. Type-2 fuzzy number

Let $\tilde{\tilde{A}}$ be a type-2 fuzzy set defined in the universe of discourse R. If the following conditions are satisfied:

(i) $\tilde{\tilde{A}}$ is normal,

(ii) $\tilde{\tilde{A}}$ is a convex set,

(iii) The support of $\tilde{\tilde{A}}$ is closed and bounded, then $\tilde{\tilde{A}}$ is called a type-2 fuzzy number.

2.3. Type-2 triangular fuzzy number

A type-2 triangular fuzzy number $\tilde{\tilde{A}}$ on R is given by $\tilde{\tilde{A}}$ = {(x,(μ_A^1(x),μ_A^2(x),μ_A^3(x)); xϵR} and $\mu_A^1(x) \leq \mu_A^2(x) \leq \mu_A^3(x)$, for all x$\epsilon$ R. Denote $\tilde{\tilde{A}} = (\tilde{A}_1, \tilde{A}_2, \tilde{A}_3)$, where $\tilde{A}_1 = (A_1^L, A_1^N, A_1^U)$, $\tilde{A}_2 = (A_2^L, A_2^N, A_2^U)$ and $\tilde{A}_3 = (A_3^L, A_3^N, A_3^U)$ are same type of fuzzy numbers.

2.3.1. Notations

$\tilde{A}^2$ = closed interval approximation, $\tilde{A}^3$ = triangular fuzzy number,
$\tilde{A}^4$ = trapezoidal fuzzy number, $\tilde{A}^5$ =piecewise quadratic fuzzy number, etc.,
$\tilde{k}^5 = (k,k,k,k,k)$, where k is constant.

2.4. Type – 2 triangular mixed fuzzy number (T2TMFN)

Let $\tilde{\tilde{A}} = (\tilde{A}_1, \tilde{A}_2, \tilde{A}_3)$ be a type – 2 triangular fuzzy number. Then there may be a choice that $\tilde{A}_1, \tilde{A}_2, \tilde{A}_3$ are different types of fuzzy numbers ie. $\tilde{\tilde{A}} = (\tilde{A}_1^3, \tilde{A}_2^4, \tilde{A}_3^2)$ or $\tilde{\tilde{A}} = (\tilde{A}_1^5, \tilde{A}_2^3, \tilde{A}_3^4)$ or $\tilde{\tilde{A}} = (\tilde{A}_1^5, \tilde{A}_2^5, \tilde{A}_3^4)$ etc. This type of type – 2 triangular fuzzy number is called type – 2 triangular mixed fuzzy number.

For a type – 2 triangular mixed fuzzy number $\tilde{\tilde{A}} = (\tilde{A}_1^3, \tilde{A}_2^4, \tilde{A}_3^2)$ with membership function $\mu_{\tilde{\tilde{A}}}(\tilde{x})$ given by

$$\mu_{\tilde{\tilde{A}}}(\tilde{x}) = \begin{cases} \frac{(\tilde{x}-\tilde{A}_1^3)}{(\tilde{A}_2^4-\tilde{A}_1^3)} & \tilde{A}_1^3 \leq \tilde{x} \leq \tilde{A}_2^4 \\ \frac{(\tilde{A}_3^2-\tilde{x})}{(\tilde{A}_3^2-\tilde{A}_2^4)} & \tilde{A}_2^4 \leq \tilde{x} \leq \tilde{A}_3^2 \\ 0 & otherwise \end{cases}$$

2.5. Ranking function

Let F(R) be the set of all type-2 normal triangular fuzzy numbers. One convenient approach for solving numerical valued problem is based on the concept of comparison of fuzzy numbers by use of ranking function. An effective approach for ordering the elements of F(R) is to define a linear ranking function $\mathbb{R}$:F(R) $\rightarrow$ R which maps each fuzzy number into R.

(i) Suppose if $\tilde{\tilde{A}} = (\tilde{A}_1^3, \tilde{A}_2^4, \tilde{A}_3^2)$, then

$\mathbb{R}(\tilde{\tilde{A}}) = (4\sum_{i=1}^{3}\tilde{A}_1^i + 3\sum_{i=1}^{4}\tilde{A}_2^i + 6\sum_{i=1}^{2}\tilde{A}_3^i)/36$.

(ii) Suppose if $\tilde{\tilde{A}} = (\tilde{A}_1^3, \tilde{A}_2^4, \tilde{A}_3^5)$, then we define

$\mathbb{R}(\tilde{\tilde{A}}) = (20\sum_{i=1}^{3}\tilde{A}_1^i + 15\sum_{i=1}^{4}\tilde{A}_2^i + 12\sum_{i=1}^{5}\tilde{A}_3^i)/180$.

(iii) Suppose if $\tilde{\tilde{A}} = (\tilde{A}_1^3, \tilde{A}_2^2, \tilde{A}_3^5)$, then we define

$\mathbb{R}(\tilde{\tilde{A}}) = (10\sum_{i=1}^{3}\tilde{A}_1^i + 15\sum_{i=1}^{2}\tilde{A}_2^i + 6\sum_{i=1}^{5}\tilde{A}_3^i)/90$.

(iv) Suppose if $\tilde{\tilde{A}} = (\tilde{A}_1^4, \tilde{A}_2^4, \tilde{A}_3^6)$, then we define

$\mathbb{R}(\tilde{\tilde{A}}) = (6\sum_{i=1}^{4}\tilde{A}_1^i + 6\sum_{i=1}^{4}\tilde{A}_2^i + 4\sum_{i=1}^{6}\tilde{A}_3^i)/72$, etc.

2.6. Arithmetic operations

Let $\tilde{\tilde{A}} = (\tilde{A}_1^3, \tilde{A}_2^4, \tilde{A}_3^2)$ and $\tilde{\tilde{B}} = (\tilde{B}_1^4, \tilde{B}_2^3, \tilde{B}_3^2)$ be two type – 2 triangular mixed fuzzy numbers then

1). Addition and subtraction:

$\tilde{\tilde{A}} \pm \tilde{\tilde{B}} = (\tilde{A}_1^3 \pm \tilde{k}^3, \tilde{A}_2^4 \pm \tilde{k}^4, \tilde{A}_3^2 \pm \tilde{k}^2)$, where k = $(4\sum_{i=1}^{4}\tilde{B}_1^i + 3\sum_{i=1}^{3}\tilde{B}_2^i + 6\sum_{i=1}^{2}\tilde{B}_3^i)/36$.

2). Scalar multiplication:

If $\alpha \geq 0$ and $\alpha \in R$, then

$\alpha\tilde{\tilde{A}} = (\alpha\overrightarrow{\tilde{A}_1^3}, \alpha\overrightarrow{\tilde{A}_2^4}, \alpha\overrightarrow{\tilde{A}_3^2})$, where $\tilde{A}_1^3 = (a, b, c)$ then $\overrightarrow{\tilde{A}_1^3} = (a, b, c)$.

If $\alpha < 0$ and $\alpha \in R$, then

$\alpha\tilde{\tilde{A}} = (\alpha\overleftarrow{\tilde{A}_1^3}, \alpha\overleftarrow{\tilde{A}_2^4}, \alpha\overleftarrow{\tilde{A}_3^2})$, where $\tilde{A}_1^3 = (a, b, c)$ then $\overleftarrow{\tilde{A}_1^3} = (c, b, a)$.

3). Multiplication:

$\tilde{\tilde{A}} \times \tilde{\tilde{B}} = \left(\frac{k'\overrightarrow{\tilde{A}_1^3}}{36}, \frac{k'\overrightarrow{\tilde{A}_2^4}}{36}, \frac{k'\overrightarrow{\tilde{A}_3^2}}{36}\right)$, where $k' = (3\sum_{i=1}^{4}\tilde{B}_1^i + 4\sum_{i=1}^{3}\tilde{B}_2^i + 6\sum_{i=1}^{2}\tilde{B}_3^i)$, $k' \geq 0$.

$$\tilde{\tilde{A}} \times \tilde{\tilde{B}} = \left(\frac{k'\overleftarrow{\tilde{A}_3^2}}{36}, \frac{k'\overleftarrow{\tilde{A}_2^4}}{36}, \frac{k'\overleftarrow{\tilde{A}_1^3}}{36}\right), k' < 0.$$

4). Division:

$\tilde{\tilde{A}} \div \tilde{\tilde{B}} = \left(\frac{36\overrightarrow{\tilde{A}_1^3}}{k'}, \frac{36\overrightarrow{\tilde{A}_2^4}}{k'}, \frac{36\overrightarrow{\tilde{A}_3^1}}{k'}\right)$, $k' \geq 0$.

$\tilde{\tilde{A}} \div \tilde{\tilde{B}} = \left(\frac{36\overleftarrow{\tilde{A}_3^2}}{k'}, \frac{36\overleftarrow{\tilde{A}_2^4}}{k'}, \frac{36\overleftarrow{\tilde{A}_1^3}}{k'}\right)$, $k' < 0$.

3. A study on arithmetic operations of type–2 triangular mixed fuzzy numbers

In this section deals with arithmetic operations of type – 2 triangular mixed fuzzy numbers using alpha cut values.

3.1. Addition of fuzzy numbers

Let $\tilde{\tilde{A}} = (\tilde{A}_1^3, \tilde{A}_2^4, \tilde{A}_3^2)$ and $\tilde{\tilde{B}} = (\tilde{B}_1^5, \tilde{B}_2^3, \tilde{B}_3^4)$ be two type – 2 triangular mixed fuzzy numbers whose membership functions are

$$\mu_{\tilde{\tilde{A}}}(\tilde{x}) = \begin{cases} \dfrac{(\tilde{x} - \tilde{A}_1^3)}{(\tilde{A}_2^4 - \tilde{A}_1^3)} & \tilde{A}_1^3 \leq \tilde{x} \leq \tilde{A}_2^4 \\ \dfrac{(\tilde{A}_3^2 - \tilde{x})}{(\tilde{A}_3^2 - \tilde{A}_2^4)} & \tilde{A}_2^4 \leq \tilde{x} \leq \tilde{A}_3^2 \\ 0 & otherwise \end{cases}$$

and $\mu_{\tilde{\tilde{B}}}(\tilde{x}) = \begin{cases} \frac{(\tilde{x}-\tilde{B}_1^5)}{(\tilde{A}_2^4-\tilde{B}_1^5)} & \tilde{B}_1^5 \leq \tilde{x} \leq \tilde{B}_2^3 \\ \frac{(\tilde{B}_3^4-\tilde{x})}{(\tilde{B}_3^4-\tilde{B}_2^3)} & \tilde{B}_2^3 \leq \tilde{x} \leq \tilde{B}_3^4 \\ 0 & otherwise \end{cases}$

Then $\alpha_{\tilde{\tilde{A}}} = \left[\left(\tilde{A}_2^4 - \tilde{A}_1^3\right)\alpha + \tilde{A}_1^3, \tilde{A}_3^2 - \left(\tilde{A}_3^2 - \tilde{A}_2^4\right)\alpha\right]$ (1)

and

$\alpha_{\tilde{\tilde{B}}} = \left[\left(\tilde{B}_2^3 - \tilde{B}_1^5\right)\alpha + \tilde{B}_1^5, \tilde{B}_3^4 - \left(\tilde{B}_3^4 - \tilde{B}_2^3\right)\alpha\right]$ (2)

are the alpha cut of type – 2 triangular mixed fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ respectively.
Adding (1) and (2) we get,

$\alpha_{\tilde{\tilde{A}}}+\alpha_{\tilde{\tilde{B}}} = [\tilde{A}_1^3 + \tilde{B}_1^5 + \left(\tilde{A}_2^4 - \tilde{A}_1^3 + \tilde{B}_2^3 - \tilde{B}_1^5\right)\alpha, \tilde{A}_3^2 + \tilde{B}_3^4 - (\tilde{A}_3^2 - \tilde{A}_2^4 + \tilde{B}_3^4 - \tilde{B}_2^3)\alpha] \cdots\cdots\cdots (*)$

To find the membership function $\mu_{\tilde{\tilde{A}}+\tilde{\tilde{B}}}(\tilde{x})$, we equate to $\tilde{x}$ in (*)

$\tilde{x} = \tilde{A}_1^3 + \tilde{B}_1^5 + \left(\tilde{A}_2^4 - \tilde{A}_1^3 + \tilde{B}_2^3 - \tilde{B}_1^5\right)\alpha$ and $\tilde{x} = \tilde{A}_3^2 + \tilde{B}_3^4 - (\tilde{A}_3^2 - \tilde{A}_2^4 + \tilde{B}_3^4 - \tilde{B}_2^3)\alpha$

Now expressing α interms of $\tilde{x}$ and setting $\alpha = 0$ and $\alpha = 1$in (*) , we get

$\alpha = \frac{\tilde{x}-(\tilde{A}_1^3+\tilde{B}_1^5)}{\left(\tilde{A}_2^4+\tilde{B}_2^3\right)-\left(\tilde{A}_1^3+\tilde{B}_1^5\right)}$; $(\tilde{A}_1^3 + \tilde{B}_1^5) \leq \tilde{x} \leq \left(\tilde{A}_2^4 + \tilde{B}_2^3\right)$ and

$\alpha = \frac{(\tilde{A}_3^2+\tilde{B}_3^4)-\tilde{x}}{(\tilde{A}_3^2+\tilde{B}_3^4)-\left(\tilde{A}_2^4+\tilde{B}_2^3\right)}$; $\left(\tilde{A}_2^4 + \tilde{B}_2^3\right) \leq \tilde{x} \leq (\tilde{A}_3^2 + \tilde{B}_3^4)$ which gives

$$\mu_{\tilde{\tilde{A}}+\tilde{\tilde{B}}}(\tilde{x}) = \begin{cases} \frac{\tilde{x}-(\tilde{A}_1^3+\tilde{B}_1^5)}{\left(\tilde{A}_2^4+\tilde{B}_2^3\right)-\left(\tilde{A}_1^3+\tilde{B}_1^5\right)} ; & (\tilde{A}_1^3 + \tilde{B}_1^5) \leq \tilde{x} \leq \left(\tilde{A}_2^4 + \tilde{B}_2^3\right) \\ \frac{(\tilde{A}_3^2+\tilde{B}_3^4)-\tilde{x}}{(\tilde{A}_3^2+\tilde{B}_3^4)-\left(\tilde{A}_2^4+\tilde{B}_2^3\right)} ; & \left(\tilde{A}_2^4 + \tilde{B}_2^3\right) \leq \tilde{x} \leq (\tilde{A}_3^2 + \tilde{B}_3^4) \end{cases}.$$

3.2. Subtraction of fuzzy numbers

Subtracting (1) and (2) we get,

$\alpha_{\tilde{\tilde{A}}}-\alpha_{\tilde{\tilde{B}}} = [\tilde{A}_1^3 - \tilde{B}_3^4 + \left(\tilde{A}_2^4 - \tilde{A}_1^3 + \tilde{B}_3^4 - \tilde{B}_2^3\right)\alpha, \tilde{A}_3^2 - \tilde{B}_1^5 - (\tilde{A}_3^2 - \tilde{A}_2^4 + \tilde{B}_2^3 - \tilde{B}_1^5)\alpha] \cdots\cdots\cdots (**)$

To find the membership function $\mu_{\tilde{\tilde{A}}-\tilde{\tilde{B}}}(\tilde{x})$, we equate to $\tilde{x}$ in (**)

$\tilde{x} = \tilde{A}_1^3 - \tilde{B}_3^4 + \left(\tilde{A}_2^4 - \tilde{A}_1^3 + \tilde{B}_3^4 - \tilde{B}_2^3\right)\alpha$ and

$\tilde{x} = \tilde{A}_3^2 - \tilde{B}_1^5 - (\tilde{A}_3^2 - \tilde{A}_2^4 + \tilde{B}_2^3 - \tilde{B}_1^5)\alpha$

Now expressing α interms of $\tilde{x}$ and setting $\alpha = 0$ and $\alpha = 1$in (**) , we get

$\alpha = \frac{\tilde{x}-(\tilde{A}_1^3-\tilde{B}_3^4)}{\left(\tilde{A}_2^4-\tilde{B}_2^3\right)-(\tilde{A}_1^3-\tilde{B}_3^4)}$; $(\tilde{A}_1^3 - \tilde{B}_3^4) \leq \tilde{x} \leq \left(\tilde{A}_2^4 - \tilde{B}_2^3\right)$ and

$\alpha = \frac{(\tilde{A}_3^2-\tilde{B}_1^5)-\tilde{x}}{(\tilde{A}_3^2-\tilde{B}_1^5)-\left(\tilde{A}_2^4-\tilde{B}_2^3\right)}$; $\left(\tilde{A}_2^4 - \tilde{B}_2^3\right) \leq \tilde{x} \leq (\tilde{A}_3^2 - \tilde{B}_1^5)$ which gives

$$\mu_{\tilde{\tilde{A}}-\tilde{\tilde{B}}}(\tilde{x}) = \begin{cases} \dfrac{\tilde{x}-(\tilde{A}_1^3-\tilde{B}_3^4)}{\left(\tilde{A}_2^4-\tilde{B}_2^3\right)-(\tilde{A}_1^3-\tilde{B}_3^4)} \; ; (\tilde{A}_1^3-\tilde{B}_3^4) \leq \tilde{x} \leq \left(\tilde{A}_2^4-\tilde{B}_2^3\right) \\ \dfrac{(\tilde{A}_3^2-\tilde{B}_1^5)-\tilde{x}}{(\tilde{A}_3^2-\tilde{B}_1^5)-\left(\tilde{A}_2^4-\tilde{B}_2^3\right)} \; ; \left(\tilde{A}_2^4-\tilde{B}_2^3\right) \leq \tilde{x} \leq (\tilde{A}_3^2-\tilde{B}_1^5) \end{cases}.$$

similarly

3.3. Multiplication of fuzzy numbers

$\mu_{\tilde{\tilde{A}}\times\tilde{\tilde{B}}}(\tilde{x}) =$

$$\begin{cases} \dfrac{-\left((\tilde{A}_2^4-\tilde{A}_1^3)\tilde{B}_1^5+(\tilde{B}_2^3-\tilde{B}_1^5)\tilde{A}_1^3\right)+\sqrt{((\tilde{A}_2^4-\tilde{A}_1^3)\tilde{B}_1^5+(\tilde{B}_2^3-\tilde{B}_1^5)\tilde{A}_1^3)^2-4(\tilde{A}_2^4-\tilde{A}_1^3)(\tilde{B}_2^3-\tilde{B}_1^5)((\tilde{A}_1^3)(\tilde{B}_1^5)-\tilde{x})}}{2(\tilde{A}_2^4-\tilde{A}_1^3)(\tilde{B}_2^3-\tilde{B}_1^5)} & \tilde{A}_1^3\tilde{B}_1^5 \leq \tilde{x} \leq \tilde{A}_2^4\tilde{B}_2^3 \\ \dfrac{\left((\tilde{B}_3^4-\tilde{B}_2^3)\tilde{A}_3^2+(\tilde{A}_3^2-\tilde{A}_2^4)\tilde{B}_3^4\right)-\sqrt{\sqrt{((\tilde{B}_3^4-\tilde{B}_2^3)\tilde{A}_3^2+(\tilde{A}_3^2-\tilde{A}_2^4)\tilde{B}_3^4)^2-4(\tilde{A}_3^2-\tilde{A}_2^4)(\tilde{B}_3^4-\tilde{B}_2^3)((\tilde{A}_3^2)(\tilde{B}_3^4)-\tilde{x})}}}{2(\tilde{A}_3^2-\tilde{A}_2^4)(\tilde{B}_3^4-\tilde{B}_2^3)} & \tilde{A}_2^4\tilde{B}_2^3 \leq \tilde{x} \leq \tilde{A}_3^2\tilde{B}_3^4 \end{cases}$$

3.4. Division of fuzzy number

$$\mu_{\tilde{\tilde{A}}\div\tilde{\tilde{B}}}(\tilde{x}) = \begin{cases} \dfrac{\tilde{x}\tilde{B}_3^4-\tilde{A}_1^3}{\left(\tilde{A}_2^4-\tilde{A}_1^3\right)+(\tilde{B}_3^4-\tilde{B}_2^3)\tilde{x}} \; ; \dfrac{\tilde{A}_1^3}{\tilde{B}_3^4} \leq \tilde{x} \leq \dfrac{\tilde{A}_2^4}{\tilde{B}_2^3} \\ \dfrac{\tilde{A}_3^2-\tilde{B}_1^5\tilde{x}}{\left(\tilde{A}_3^2-\tilde{A}_2^4\right)+\left(\tilde{B}_2^3-\tilde{B}_1^5\right)\tilde{x}} \; ; \dfrac{\tilde{A}_2^4}{\tilde{B}_2^3} \leq \tilde{x} \leq \dfrac{\tilde{A}_3^2}{\tilde{B}_1^5} \end{cases}$$

4. Conclusion

In this paper, presents addition, subtraction, multiplication and division of type -2 triangular mixed fuzzy numbers using alpha cut methods. In future inverse, exponential and logarithm etc., of type – 2 triangular mixed fuzzy numbers can be studied.

REFERENCES

1. R.I.John, Type – 2 fuzzy sets, An appraisal of theory and applications, Int. J. Fuzziness Knowledge - Based Systems, 6(6) (1998) 563-576.
2. K.Latha, K.Thiripurasundari and R.Arulmozhi, A study on scalar triangular type – 2 triangular fuzzy matrices, Global Journal of Pure and Applied Mathematics, 13(2) (2017) 193-198.
3. M.Manvizhi, K.Latha and K.Thiripurasundari, Solving system of simultaneous type – 2 fuzzy linear equations, Global Journal of Pure and Applied Mathematics, 13(2) (2017) 193-198.
4. S.Mahanta, R.Chutia and H.K.Barush, Fuzzy arithmetic with and without using the method of alpha cut, International journal of latest trends in computing, 1(2) (2010) 73-80.
5. D.S.Dinagar and A.Anbalagan, Fuzzy programming based on type-2 Generalized fuzzy number, International J. Math. Sci. & Engg. Appls., 5(4) (2011) 317-329.
6. K.Thiripurasundari, A study on type – 2 triangular mixed fuzzy numbers, International Journal of Scientific Research in Science, Engineering and Technology, 2(5) (2016) 83-85.
7. K.Thiripurasundari, K.Latha and S.Deepika, A study on different types of type-2 mixed fuzzy numbers, Global Journal of Pure and Applied Mathematics, 13(2) (2017) 193-198.
8. L.A.Zadeh, Fuzzy sets, Information and Control, 8 (1965) 338-353.

9. L.A.Zadeh, The fuzzy concept of a linguistic variables and its application to approximate reasoning, 1, Information Sciences, 8 (1975) 199 – 249.