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# Topological Structures on d-Algebras

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**ABSTRACT**

There has been some recent interest in applying topological structure on BCH algebras. In this paper Lee and Ryu initiates the study of topological BCK algebras. Motivated by this, in this paper we define the notion of topological d-algebras, give some examples and prove some important theorems.

***Keywords:*** d-algebra, topological d-algebra, topological d-sub algebra, topological d-ideal.

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## 1. Introduction

Imai and Iseki introduced two classes of abstract algebras BCK algebras and BCI-algebras [5, 6, 7, 8]. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. In [3, 4] Hu and Li introduced a wider class of abstract algebras BCH algebras. They have shown that the class of BCI algebras is a proper subclass of the class of BCH-algebras. Neggers and Kim introduced d algebras [12]. Dar introduced the notions of left and right mapping on BCK-algebras in [1]. Further in [10] the authors discussed notions of endomorphism's on BCH-algebras. Lots of paper have been published in related algebraic structure [7-19]. Left and Right mapping over BCI-algebras have been discussed in [2]. In [9] the author studied topological structure on BCH algebras and proved some theorems that determine the relationship between them. There has been some recent interest in applying topological notions to non mainstream algebras. Motivated by this, in this paper, we study the connection between topology and d-algebras.

## 2. Preliminaries

**Definition 2.1.** [1, 3, 4] A $BCH$-algebra $(X,*,0)$ is a non empty set $X$ with a constant $0$ and a binary operation satisfying the following conditions.

1. $x * x = 0$
2. $(x * y) * z = (x * z) * y$

3. $x * y = 0 \; and \; y * x = 0 \Rightarrow x = y \; \forall \; x, y, z \in X.$

**Definition 2.2.** [8] Let $X$ be a set with a binary operation $*$ and a constant 0. Then $(X,*,0)$ is called a $BCI$-algebras if it satisfies the following axioms.

1. $\big((x * y) * (x * z)\big) * (z * y) = 0$
2. $\big(x * (x * y)\big) * y = 0$
3. $x * x = 0$
4. $x * y = o$ and $y * x = 0 \Rightarrow x = y \; \forall \; x, y \in X.$

**Definition 2.3.** [2, 7] Let X be a set with a binary operation $*$ and a constant 0. Then $(X,*,0)$ is called a $BCK$-algebras if it satisfies the following axioms.

1. $\big((x * y) * (x * z)\big) * (z * y) = 0$
2. $\big(x * (x * y)\big) * y = 0$
3. $x * x = 0$
4. $x * y = 0$ and $y * x = 0 \Rightarrow x = y \; \forall \; x, y, z \in X$
5. $0 * x = 0.$

**Definition 2.4.** [12] A $d$-algebra is a non empty set $X$ with a constant $0$ and binary operation $*$ and satisfying the following axioms.

1. $x * x = 0$
2. $0 * x = 0$
3. $x * y = 0$ and $y * x = 0 \Rightarrow x = y \; \forall \; x, y \in X.$

**Example 2.5.** Let $(X = \{0,1,2,3\},*,0)$ be a set with the following cayley table.

| * | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 2 | 2 | 2 | 0 | 0 |
| 3 | 3 | 3 | 1 | 0 |

Then $(X,*,0)$ is a $d$-algebra.

**Definition 2.6.** [11] A topology, on a $BCK$-algebra $X$ is $BCK$-algebra topology and $X$ furnished with, is a topological $BCK$-algebras if $(x, y) \rightarrow x * y$ is continuous from $X \times X$ with its Cartesian product topology to $X$ with the topology. In this case abbreviate we call $X$ a $TBCK$-algebra.

**Definition 2.7.** [9] Let $(X,*)$ be a $BCH$-algebra and $\tau$ a topology on $X$. Then $X = (X,*,\tau)$ is called a topological $BCH$-algebra, if the operation $*$ is continuous, or equivalently, for any $x, y \in X$ and for any $x, y \in X$ and for any open set $W$ of $x * y$ there exist two open sets $U$ and $V$ respectively such that $U * V$ is a subset of $W$.

**Definition 2.8.** [1] Let $X$ be a $BCH$-algebra. If $X$ is satisfies the condition,

$(x * y) * z = (x * y) * (y * z), \forall\, x, y, z \in X$. Then $X$ is called positive implicative $BCH$-algebra.

**Definition 2.9.** [10] Let $(X,*)$ be a $BCH$-algebra, and $a \in X$. Define a left map $L_a : X \to X$ by, $L_a(x) = a * x \;\forall\, x \in X$.

Let $(X,*)$ be a $BCH$-algebra, and $a \in X$. Define a right map $R_a : X \to X$ by, $R_a(x) = x * a \;\forall\, x \in X$.

Let $(X,*)$ be a $BCH$-algebra. The set of all left maps on $X$ is defined as, $L(X)$.

## 3. Topological structures on d-algebras

**Definition 3.1.** Let $X$ be a $d$-algebra and $U, V$ are any non empty subsets of X. We define a subset $\boldsymbol{U * V}$ of $X$ as follows. $U * V = \{x * y \mid x \in U,\; y \in V\}$

**Example 3.2.** Let $X = (\{0, a, b, c\}, * ,0)$ be a $d$-algebra the binary operation * will be defined as follows.

.

| * | 0 | a | b | c |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| a | a | 0 | b | b |
| b | b | b | 0 | 0 |
| c | c | c | a | 0 |

Let $U = \{0, a\}$ and $V = \{a, b\}$ be two subsets of $X$.

$U * V = \{0 * a, 0 * b, a * a, a * b\}$

$= \{0,0,0, b\}$

$= \{0, b\}$

subsets of $X$.

**Definition 3.3.** Let $(X,*)$ be a $d$-algebra. Let $\tau$ be the collection of subsets of $X$. $\tau$ is said to be a topology on $X$. If

1. $X, \emptyset \in \tau$.
2. arbitrary union of members of $\tau$ is in $\tau$.
3. finite intersection of members of $\tau$ is in $\tau$.

**Definition 3.4.** Let $(X,*)$ be a $d$-algebra and $\tau$ a topology on $X$. Then $X = (X,* ,0, \tau)$ is called a **topological $\boldsymbol{d}$-algebra**, **(it is denoted by $\boldsymbol{Td}$-algebra)** if the operation " $*$ " is continuous or equivalently for any $x, y \in X$ and for any open set $W$ of $x * y$ there exist two open sets $U$ and $V$ respectively such that $U * V$ is a subset of $W$.

**Example 3.5.** Let $(X = \{0,1,2\}, * ,0)$ be a $d$-algebra the binary operation * will be defined as follows:

| * | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 2 | 0 | 2 |
| 2 | 1 | 1 | 0 |

Define a topology $\tau = \{\emptyset, \{0\}, \{1\}, \{0,1\}, X\}$. Let $x = 2,0$ and $y = 1,2$ then

$x * y = \{2 * 1, 2 * 0, 0 * 1, 0 * 2\} = \{1,0,0,0\} = \{0,1\} = W$ . Put $U = \{0\}$ and $V = \{1\}$,

$U * V = \{0 * 1\} = \{0\} \subseteq W$. Therefore $\{0\} \subseteq \{0,1\}$. Hence $(X, *, 0, \tau)$ is a $Td$-algebra.

**Example 3.6.** Let $(X = \{0,1,2\}, *, 0)$ be a $d$-algebra the binary operation $*$ will be defined as Define a topology $\tau = \{\emptyset, \{0\}, \{1\}, \{0,1\}, X\}$.

| * | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 2 | 0 | 2 |
| 2 | 1 | 1 | 0 |

Let $x = 2$ and $y = 1$ then $x * y = 2 * 1 = \{1\} = W$.

Put $U = \{0\}$ and $V = \{1\}, U * V = \{0 * 1\} = \{0\} \not\subset W$.

Therefore $(X, *, 0, \tau)$ is not a $Td$-algebra.

**Remark 3.7.** The member of $\tau$ are called an open set in $X$. The complement of $A \in \tau$, that is $X \backslash A$ is called a closed set in $X$, denoted by $\breve{A}$. The open set of an element $x \in X$, is a member of $\tau$ containing $X$.

**Definition 3.8.** Let $X$ and $Y$ be a $Td$-algebra. A map $f: X \to Y$ is continuous, if for every $x \in X$ and any open set $W$ of $f(x)$, there exist an open set $V$ of $x$ such that $f(V)$ subset of $W$.

**Example 3.9.** Let $(X = \{0,1,2,3\}, *, 0, \tau)$ and $(Y = \{0,1,2\}, *, 0, \tau)$ are the $d$-algebra the binary operation $*$ will be defined an $X$ and $Y$ are as follows respectively. The topology $\tau_1 = \{\emptyset, \{1,0\}, \{2,3\}, X\}$ is on $X$ and the topology $\tau_2 = \{\emptyset, \{2\}, \{1,0\}, X\}$ is on $Y$.

| * | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 3 | 2 |
| 2 | 2 | 3 | 0 | 1 |
| 3 | 3 | 2 | 1 | 0 |

| * | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 2 |
| 2 | 2 | 2 | 0 |

The map $f: X \to Y$ is defined by, $f(x) = 1$, for all $x \in X$ is continuous.

**Definition 3.10.** Let $X$ be a $Td$-algebra, and $x \in X$. Define a **left map** $L_a: X \to X$ by $L_a(x) = a * x, \forall\ a \in X$.

**Example 3.11.** Let $(X = \{0,1,2\}, * ,0, \tau)$ be a $Td$-algebra the binary operation * will be defined as follows,

| * | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 2 | 0 | 2 |
| 2 | 1 | 1 | 0 |

$\tau = \{\emptyset, \{0,1\}, \{2,3\}, X\}$ is a topology on $X$. For $x \in X$, the map $L_2: X \to Y$ is a left map defined by, $L_2(X) = 2 * X$.

**Theorem 3.12.** Let $X$ be a $Td$-algebra. Every left map on $X$ is continuous.

Proof. Let $x \in X$. Define a left map $L_x: X \to X$ by, $L_a(y) = x * y\ \forall\ y \in X$. Since $X$ is $Td$- algebra, for every$x, y \in X$ and any open set $W$ of $x * y$, there exist a two open sets $U$ and $V$ of $x$ and $y$ respectively, such that $U * V$ is a subset of $W$.

Clearly, $x * V \subseteq U * V$. $L_x(V)$ subset of $W$. Since $x$ is arbitrary, $L_x$ is continuous.

**Definition 3.13.** Let $X$ be a $Td$-algebra. If $X$ satisfies the condition,

$(x * y) * z = (x * y) * (y * z)$, for all $x, y, z \in X$. Then $X$ is called **positive implicative $Td$-algebra**.

**Definition 3.14.** Let $X$ be a positive implicative $Td$-algebra and $A$ be any subset of $X$. Define $L_A = \{L_A | a \in A\}$.

**Definition 3.15.** Let $X$ be a $Td$-algebra, define a map $\varphi: X \to L(X)$ by $\varphi(x) = L_x$.

**Remark 3.16.** Let $X$ be a positive implicative $Td$-algebra, and $A, B$ be any subset of $X$. If $A \subseteq B$ then $\varphi(A) = \varphi(B)$. Let $X$ be a positive implicative $Td$-algebra and $G_1, G_2$ any subset of $X$. If $G = G_1 \cup G_2$ then $\varphi(G) = \varphi(G_1) \cup \varphi(G_2) = \varphi(G_1 \cup G_2)$.

**Theorem 3.17.** The map $\varphi$ is a $Td$-isomorphism.

Proof. Clearly, $\varphi$ is 1-1 and onto. Consider, $\varphi(x*y) = L_{(x*y)}$ imply

$L_{x*y}(t) = (x*y)*t = (x*t)*(y*t) = \varphi(x)*\varphi(y), \forall\, x, y \in X.$

Hence $\varphi$ is a $Td$-isomorphism.

**Definition 3.18.** Let $(X,*,0,\tau)$ be a positive implicative $Td$-algebra, and the collection of subsets of $L(X), \tau' = \{L_G \subseteq L(x) \mid G \in \tau\}$ is a called a $\boldsymbol{L(S)}$**-topology** on the set $L(X)$.

**Example 3.19.** Let $(X = \{0,1,2,3\},*,0,\tau)$ be a positive implicative $Td$-algebra the binary operation $*$ will be defined as follows.

| * | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 1 |
| 2 | 2 | 2 | 0 | 2 |
| 3 | 3 | 3 | 3 | 0 |

$\tau = \{\emptyset, \{2\}, \{3\}, \{0,1\}, \{2,3\}, \{0,1,3\}, \{0,1,2\}, X\}$ is a topology on $X$, Now,

$L(X) = \{L_0, L_1, L_2, L_3\}$ and the $LS$-topology.

$$\tau' = \{\emptyset, \varphi(\{2\}), \varphi(\{3\}), \varphi(\{0,1\}), \varphi(\{2,3\}), \varphi(\{0,1,3\}), \varphi(\{0,1,2\}), L(X)\}$$

**Theorem 3.20.** Let $(X,*,0,\tau)$ be a positive implicative $Td$-algebra, then $\tau' = \{\varphi(G) \mid G \in \tau\}$ is a topology on $L(X)$.

Proof. It is trivial that, $L(X)$ and $\varphi \in \tau'$ .Let $\{\varphi(G_i)\} \in \tau'$, implies that $\{G_i\} \in \tau$.Then $U_{i\in I} G_i \in \tau, U_{i\in I}\varphi(G_i) \in \tau'$, similarly, we can prove a finite intersection of element of $\tau'$ is in $\tau'$.

**Definition 3.21.** Let $S$ be a non empty subset of a $Td$-algebra $X$, then $S$ is called $\boldsymbol{Td}$**-subalgebra** of $X$ if $x*y \in S\ \forall\, x, y \in S$.

**Example 3.22.** Let $X = \{0,1,2,3\}$ be a $Td$-algebra in which the operation $*$ is defined as follows:

| * | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 2 | 2 | 2 | 0 | 0 |
| 3 | 3 | 3 | 1 | 0 |

The topology $\tau = \{\emptyset, \{0\}, \{1\}, \{2\}, \{1,2\}, \{1,0\}, \{2,0\}, \{1,2,0\}, X\}$.

Let $x = \{1,2\}, y = \{2,3\},\ \ x, y \in X$,

$$x * y = \{(1 * 2), (1 * 3), (2 * 2), (2 * 3)\}$$
$$= \{1,0,0,0\}$$
$$= \{1,0\}$$
$$= W,$$

Let $U = \{1,0\}$ and $V = \{2\}$,

clearly, $U * V = \{(1 * 2), (0 * 2)\}$

$= \{1,0\} \subseteq W$

Therefore $(X,* ,0, \tau)$ is a $Td$-algebra.

In $X$ the sets $S_1 = \{0,1,2\}$, $S_2 = \{0,1\}, S_3 = \{0,1,3\} and\ S_4 = \{0,2\}$ are $Td$-subalgebra of $X$, while $S = \{0,2,3\}$ is not a $Td$-subalgebra of $X$.

**Definition 3.23.** Let $X$ be a $Td$-algebra and $I$ be a subset of $X$, then $I$ is called a $\boldsymbol{Td}$**-ideal** of $X$. If it satisfies the following conditions.

1.$0 \in I$
2.$x * y \in I$ and $y \in I \Rightarrow x \in I$
3.$x \in I$ and $y \in X \Rightarrow x * y \in I$ $(i.e)$ $I * x \subseteq I$

**Example 3.24**. Let $X = \{0,1,2,3\}$ be a $Td$-algebra in which the operation * is defined as follows.

| * | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 2 | 2 | 2 | 0 | 0 |
| 3 | 3 | 3 | 1 | 0 |

In $X$ the sets $I_1 = \{0,1\}$ and $I_2 = \{0,2\}$ are $Td$-ideals of $X$. While $I_4 = \{1,2\}$ are not $Td$-ideals of $X = \{0,1,2,3\}$.

**Remark 3.25.** Let $(X,*,0,\tau)$ be a $Td$-algebra. Every $Td$-ideal of $X$ is a $Td$-subalgebra of $X$ but the converse need not be true. For the example, Let $X = \{0, a, b, c\}$ is a $Td$-algebra the binary operation $*$ will be defined as follows,

| * | 0 | a | b | c |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| a | a | 0 | 0 | b |
| b | b | c | 0 | 0 |
| c | c | c | c | 0 |

$I = \{0, a\}$ is a $Td$-subalgebra but not a $Td$-ideal of $X$ for $a * c = b \notin I$.

**Definition 3.26.** Let $(X,*,0,\tau)$ be a $Td$-algebra define a binary relation $\leq$ on $X$ by taking $x \leq y$ if and only if $x * y = 0$. In this case $(X, \leq)$ is a **partially ordered set**.

**Definition 3.27.** Let $(X,*,0,\tau)$ be a $Td$-algebra and $x \in X$ define $x * X = \{x * a | a \in X\}$. $X$ is said to be **edge $Td$-algebra** if for any $x \in X$, $x * X = \{x, 0\}$.

**Example 3.28.** Let $X = \{0,1,2\}$ be a set with the following cayley table.

| * | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 |
| 2 | 2 | 2 | 0 |

(X, *) is a edge $Td$-algebra.

**Lemma 3.29.** Let $(X,*,0,\tau)$ be an edge $Td$-algebra, then $x * 0 = x$ for any $x \in X$.

Proof. Since $(X,*,0,\tau)$ is an edge $Td$-algebra, either $x * 0 = x \, or \, x * 0 = 0$ for any $x \in X$. Let $X \neq 0$ and $x * 0 = 0$. By condition two of the $d$-algebra $0 * x = 0$. Thus we have $x * 0 = 0$ and $0 * x = 0$. Hence by the condition three of the $d$-algebra $x = 0$ a contradiction to the fact that $x \neq 0$. Hence we have $x * 0 = x \; \forall \; x, y \in X$.

**Proposition 3.30.** If $(X,*,0,\tau)$ is an edge $Td$-algebra, then the condition

$\big(x * (x * y)\big) * y = 0 \; \forall \; x, y \in X$ holds.

**Proof.** If $x = 0$ then $\big(x * (x * y)\big) * y = 0$ (since $(0 * x = 0)$). Let $x \neq 0$, $x * y \in X$. Since $x * y = x \; or \; 0$, Now $x * y \neq 0, x * y = x$. $\big(x * (x * y)\big) * y = (x * x) * y = 0$.

**Definition 3.31.** A $Td$-algebra $X$ is said to be **commutative** if $x \wedge y = y \wedge x$ for all $x, y \in X$.

**Example 3.32.** Let $X = \{0,1,2\}$ be a $Td$-algebra which the operation $*$ is defined as follows,

| * | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 |
| 2 | 2 | 1 | 0 |

For $1 \wedge 2 = 2 * (2 * 1) = 2 * 1 = 1 \quad \rightarrow (1)$

$2 \wedge 1 = 1 * (1 * 2) = 1 * 0 = 1 \quad \rightarrow (2)$

From (1) and (2), ( $X$,*) is a commutative $Td$-algebra.

**Example 3.33**. Let $X = \{0,1,2\}$ be a $Td$-algebra which the operation $*$ is defined as follows,

| * | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 |
| 2 | 2 | 2 | 0 |

( $X$,*) is a commutative $Td$-algebra.

**Example 3.34.** Let $X = \{0,1,2,3\}$ be a $Td$-algebra which the operation $*$ is defined as follows,

| * | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 1 |
| 2 | 2 | 2 | 0 | 0 |
| 3 | 3 | 3 | 3 | 0 |

For $1 \wedge 2 = 2 * (2 * 1) = 2 * 2 = 0 \quad \rightarrow (1)$

$$2 \wedge 1 = 1 * (1 * 2) = 1 * 0 = 1 \quad \rightarrow (2)$$

From $(1) \neq (2)$.

( $X$,*) is not a commutative $Td$-algebra.

**Definition 3.35.** A $Td$-algebra $(X,*,0,\tau)$ is said to be $\boldsymbol{Td}$**-transitive** if $x * y = 0$ and $z * y = 0 \Rightarrow x * y = 0$.

**Example 3.36.** Consider the following $Td$-algebra $X$ with the table.

| * | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 |
| 2 | 2 | 2 | 0 | 0 |
| 3 | 3 | 3 | 3 | 0 |

We can easily see that $1 * 2 = 0$, $2 * 3 = 0$ but $1 * 3 = 0$ and hence $(X,*,0,\tau)$ is a $Td$-transitive.

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