2018

M.Sc.

Part–II Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—IX (OR/OM)

Full Marks : 100

Time : 4 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Special Paper : OR

(Advanced Optimization and Operations Research–I)

Answer Q. No. *11* and any *six* from the rest.

1. (a) State and prove Fritz-John saddle point necessary optimality theorem. When does the theorem fail? 6+2

(b) Give the outlines of the decomposition principle to solve a large linear programming problem. 8

2. (a) Lex X° be an open set in R^n, let θ and g be defined on X°. Find the conditions under which.

(i) a solution $(\bar{x}, \bar{r}_0, \bar{r})$ of the Fritz-John saddle point problem is a solution of the Fritz-John stationary point problem and conversely.

(ii) a solution $(\bar{x}, \bar{u})$ of the Kuhn-Tucker saddle point problem is a solution of the Kuhn-Tucker stationary point problem and conversely. 4

(b) Briefly describe the Wolfe's method to solve a quadratic programming problem. 6

(c) State Farkas' theorem. Give the geometric interpretation of it. 6

3. (a) Solve the following quadratic programming problem by Beale's method

Max $Z(x_1, x_2) = 6x_1 + 3x_2 - x_1^2 - 4x_2^2 + 4x_1x_2$

Subject to

$x_1 + x_2 \le 3$

$4x_1 + x_2 \le 9$

$x_1, x_2 \le 0$

Also show that $Z(x_1, x_2)$ is strictly convex. 8+2

(b) What do you mean by the theorem of alternative? State and prove Motzkin's theorem of alternative. 2+4

4. (a) Define constraint qualification and explain the cause of their introduction in the theory of non-linear programming problem. Discuss about two such constraint qualifications. 8

(b) Define goal programming problem. A firm produces two products A and B. Each product must be processed through two departments namely 1 and 2. Department 1 has 30 hours of production capacity per day, and department 2 has 60 hours. Each unit of product A requires 2 hours in department 1 and 6 hours in department 2. Each unit of product B requires 3 hours in department 1 and 4 hours in department 2. Management has established the following goals it would like to achieve in determining the daily product mix :

P_1 : The joint total production at least 10 units.

P_2 : Producting at least 7 units of product B.

P_3 : Producing at least 8 units of product A.

Formulate this problem as a goal programming model. 3+5

5. (a) Prove that the quadratic function $Q(X)=\frac{1}{2}X^{T}AX+B^{T}X+C$, where A be $n \times n$ symmetric matrix, $B, X \in R^{n}$ and C is a real constant, is minimized sequentially once along each direction of a set of n A-conjugate directions, then the global minimum $Q(X)$ will be located before the nth step regardless of the starting point and the order in which the direction is used. 8

(b) Use the artificial constraint method to find the initial basic solution of the following problem and them apply the dual simplex method to solve it.

Max $z=2x_1-3x_2-2x_3$

Subject to $x_1-2x_2-3x_3=8$

$2x_2+x_3 \le 10$

$x_2-2x_3 \ge 4$

$x_1,\ x_2,\ x_3 \ge 0$ 8

6. (a) Solve the following IPP using Gomory's cutting plane method

Maximize $z=7x_1+9x_2$

Subject to the constraints

$-x_1+3x_2 \le 6$

$7x_1+x_2 \le 35$

$x_1+x_2 \ge 0$ and integers. 8

(b) What is usefulness of post optimality analysis? Derive the conditions of the range of discrete changes of cost vector (C) of the LPP

Maximize $Z=CX$

subject to the constraints

$AX=b$

and $X \ge 0$ 2+6

7. (a) Use Davidon-Fletcher-Powell method for

Min $f(x_1,x_2)=8x_1^2+4x_2^2-24x_1+16x_2+35$

With $(0.5,1)^T$ as the starting point. 8

(b) State and prove Tucker's lemma on non-linear programming. 8

8. (a) Minimize $f(x)=\begin{cases}\dfrac{x^2-6x+13}{4} & x\le 4\\ x-2 & x>4\end{cases}$

in the interval [2, 5] by Fibonacci method by taking $n = 5$ 8

(b) Using bounded variable technique solve the following LPP

Maximize $z = 6x_1 - 2x_2 - 3x_3$

Subject to the constraints

$2x_1 + 4x_2 + 2x_3 \le 8$

$x_1 - 2x_2 + 3x_3 \le 7$

$0 \le x_1 \le 2$

$0 \le x_2 \le 2$

and $0 \le x_3 \le 1$ 8

9. (a) Use revised simplex method to solve the LPP

Maximize $z = 3v_1 + 2v_2 + 5v_3$

Subject to $v_1 + 2v_2 + v_3 \le 430$

$3v_1 + 2v_3 \le 460$

$v_1 + 4v_2 \le 420$

$v_1,\ v_2,\ v_3 \ge 0$ 8

(b) What is mixed integer programming problem? Give two real examples of this type. Describe branch and bound method to solve an integer programming problem. 2+1+5

10. (a) Consider the final table of an LPP :

C_B	Y_B	Y_B	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7	Y_8
2	Y_1	3	1	0	0	−1	0	5	2	−1
4	Y_2	1	0	1	0	2	1	−1	0	5
1	Y_3	7	0	0	1	−1	−2	5	−3	2
		$Z_j - C_j$	0	0	0	2	0	2	1	2

Where Y_6, Y_7 *and* Y_8 are slack variables.

If the constraint

$2x_1 + 3x_2 - x_3 + 2x_4 + 4x_5 \le 1$ is added then determine the solution of the changed LPP. 8

(b) Let X_{B_r} be a negative basic variable in a dual simplex table and all net evaluations $z_j - c_j \geq 0$ and the primal LPP is of maximiation. If $y_{rj} \geq 0$ for all non-basic variables x_j, then show that there does not exist any feasible solution to the primal LPP. 8

11. Write a short note on any *one* of the following : 1×4

(a) Unimodal function and convex programming.

(b) Complementary slackness conditions.

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Special Paper : OM

Answer Q. No. *1* and any *six* from the rest.

1. Answer any *two* questions : 2×2

(a) Write the continuity equation for Quasi-Geostrophic flow.

(b) Explain "stationary wave".

(c) Define Reynolds number and discuss its physical significance for the case of it higher value.

2. Obtain the Gibb's general thermodynamical relation for sea-water. Hence, deduce Gibb's-Duhem relation. Write the physical significance of Gibb's relation. 8+5+3

3. (a) Derive the following relations : 8+8

(i) $\Gamma = \frac{T}{c_p}\left(\frac{\partial v}{\partial T}\right)$

(ii) $c^2 = \frac{1}{\rho k_n}$

(b) Show that $\eta dT - v dp - (1-s)d\mu + d\mu_s = 0$.

4. Show that the necessary condition of thermodynamical equilibrium of a finite volume of sea-water are

$$T = \frac{-1}{\lambda},\ \mu_s = -U - \frac{\lambda_s}{\lambda} + \frac{q^2}{2},$$

$$\mu_w = -U - \frac{\lambda_w}{\lambda} + \frac{q^2}{2},$$

$$\vec{q} = -\frac{\vec{a}}{\lambda} - \frac{\vec{b} \times r}{\lambda}$$

Where symbols have their usual meanings. 16

5. Define stratified fluid. Discuss Brunt-Vaisala frequency. Express this frequency in term of T, c_p and c_v.

6. State the assumption of Boussinesq's approximation. Hence, derive field equations approximately accordingly Boussinesq's approximation. 16

7. (a) Establish the equations of small amplitude oceanic wave motion on a rotating earth.

(b) Define salinity (s) and concentration of pure water (c_w) and hence, show that $s + c_w = 1$. 8+8

8. Define Rossby number. Deduce the governing equations of thermal wind when Rossby number is small. Hence, deduce the Taylor-Proudman theorem. 2+10+4

9. (a) Derive the kinematical and pressure condition of the free surface for the progressive wave. 8+8

(b) Derive an expession for speed of propagation of a progressive wave in the surface of a canal of finite depth. Hence show that for large wave lengh the speed of propagation tends to $\sqrt{gh}$, where h is the depth of the canal.

10. Define kinetic and potential energy. Prove that the total energy of progressive wave is $\frac{1}{2}\rho g a^2 \lambda$ where a, λ are the wave amplitude and wave length respectively. 2+2+12