2018

M.Sc.

Part-II Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—VIII

Full Marks : 100

Time : 4 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Group—A

Answer Q. No. *1* is compulsory and any *three* from the rest.

1. Answer any *two* questions : 2×4

(a) (i) For each of the following functions, determine which has a Laplace transform. If it exists, find it, if does not satisfy, why [a] $1/t$, [b] $\ln(t)$. 2

(ii) Define the term convolution on Fourier transform. 2

(b) (i) Define the inversion formula for Fourier cosine transform of a function $f(x)$. What happens if $f(x)$ is continuous ? 2

(ii) Define an eigen value and eigen function of an integral equation. 2

(c) (i) Verify the final value theorem in connection with Laplace transform of the function $t^3 e^{-t}$. 2

(ii) Define Hankel transform of order n of a function $f(r), 0 \le r \le a$ and state its inversion formula. 2

2. (a) Form an integral equation corresponding to the differential equation $\frac{d^2y}{dx^2} = -\lambda y(x)$, with the condition $y(0) = 0$, $y(1) = 0$ and find its kernel. 7

(b) Find the Laplace transform of the function : $\frac{e^{-t} - e^{-3t}}{t}$. 3

(c) Using Green's function method, solve the following differential equation $y'''(x) = 1$, subject to boundary conditions $y(0) = y(1) = 0, y'(0) = y'(1)$. 6

3. (a) State and prove Parseval's identity on Fourier transform. 6

(b) With the help of the resolvent kernel, find the solution of the integral equation

$$y(x) = 1 + x^2 + \int_0^x \left(\frac{1+x^2}{1+t^2} \right) y(t)\,dt$$ 6

(c) Prove that the product of two functions is a good function where one function is good function and other one is fairly good function. 4

4. (a) Find the solution of the problem of free vibration of a stretched string of infinite length PDE :

$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = 0, -\infty < x < \infty,$$

with boundary conditions $u(x, 0) = f(x)$, $-\infty < x < \infty$, $\frac{\partial u(x,0)}{\partial t} = g(x)$ and u and $\frac{\partial u}{\partial x}$ both vanish as $|x| \to \infty$. 8

(b) Find the value of $\sin(t) * t^2$ where $*$ denotes the convolution operator on Laplace transform. 4

(c) Prove that the integral of a good function is not necessarily a good function. 4

5. (a) If a and b are real constants, solve the following integral equation, $ax+bx^2=\int_0^x \frac{y(t)}{(x-t)^{1/2}}dt.$ 7

(b) If a real valued function $f(t)$ of real variable which is piecewise continuous in any finite interval of t and is exponential order $O(e^{vt})$ as $t \to \infty$, when $t \geq 0$ then prove that the integrals $\int_0^\infty f(t)e^{-pt}dt$, converges in the domain Real $(p) > v$. 6

(c) Find the zero-order Hankel transform of $f(r)=H(a-r), H(r)$ stands for Heaviside step function. 3

6. (a) Use the Laplace transformation technique to solve the differential equation :

$$\frac{d^2x}{dt^2}+3\frac{dx}{dt}+2x=2t-e^{-t}, t>0,$$

which satisfies $x(0)=\frac{1}{2}, \frac{dx}{dt}=0$ at $t>0$. 7

(b) Prove that the Fourier transform of $\frac{1}{x}$ is $i\sqrt{\frac{\pi}{2}}\,\mathrm{sgn}(x)$, where sgn$(x)$ is a signum function.

(c) All the eigen values of regular SL problem with $r(x) > 0$, are real. 5

Group—B (OR)

(Elements of Optimization and Operations Research)

[Marks : 50]

Answer Q. No. 7 and any *three* from the rest.

7. What do you mean by EOQ ? 2

Or

Define Convex and Concave functions. 2

8. (a) Using Bellman's principle of optimality, solve the following problem

Minimize $z = y_1 + y_2 + y_3 + \cdots + y_n$.

Subject to the constraints $y_1 \cdot y_2 \cdot y_3 \cdots y_n = b$
$y_i \geq 0; \quad i = 1, 2, \ldots n.$

(b) Solve the following LPP by revised simplex method 8

Minimize $Z = 2x_1 + x_2$

subject to constraints

$$3x_1 + x_2 \le 3$$
$$4x_1 + 3x_2 \ge 6$$
$$x_1 + 2x_2 \le 3$$

and $x_1, x_2 \ge 0.$

9. (a) Find the optimal order quantity for a product when the annual demand for the product is 500 units, the cost of storage per unit per year is 10% of the unit cost and ordering cost per order is Rs. 180. The unit costs are given below : 8

Quantity	*Unit Cost (Rs.)*
$0 \le q_1 < 500$	25.00
$500 \le q_2 < 1500$	24.80
$1500 \le q_3 < 3000$	24.60
$3000 \le q_4$	24.40

(b) Derive the conditions for the range of discrete changes of cost vector (C) of the LPP 8

Maximize $Z = CX$

Subject to $AX = b$ and $X \ge 0$

such that the optimal basic feasible solution does not changed.

10. (a) Solve the following IPP by Gomory's cuting plane method 8

Minimize $z = 2x_1 + 3x_2$

subject to constraints $80x_1 + 31x_2 \ge 248$

$x_1, x_2 \ge 0$ and are integers.

(b) Briefly describe the Wolfe's method to solve a quadratic programming problem. 8

11. (a) Use Beale's method for solving the quadratic programming problem :

Maximize $f = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$

subject to the constraints $x_1 + 2x_2 \le 2$ and $x_1, x_2 \ge 0.$ 8

(b) Derive the optimal order quantity expression of a multi-items inventory model without shortages when the amount of investment is given. 8

12. (a) Solve the following LPP be dynamic programming method 8

Maximize $Z = 8x_1 + 7x_2$

subject to the constraints

$$2x_1 + x_2 \le 8$$
$$5x_1 + 2x_2 \le 15$$
and $$x_1, x_2 \ge 0.$$

(b) Solve the following non-linear programming problem given below :

Optimize $Z = x_1^2 + x_2^2 + x_3^2$ 8

subject to the constraints 8

$$x_1 + x_2 + 3x_3 = 2$$
$$5x_1 + 2x_2 + x_3 = 5$$
$$x_1, x_2, x_3 \ge 0.$$

Group—B (OM)

(Dynamical Oceanology and Methodology)

[Marks : 50]

Answer Q. No. *12* and any *three* from the rest.

7. (a) Show that the sum of kinetic energy, potential energy and enthalpy of an air parcel in the atmosphere

remains constant when the flow is steady, adiabatic and frictionless. 8

(b) Deduce Gibb's general tthermodynamical relation for sea-water. Hence, derive Gibb's-Duhem relation. 8

8. (a) Derive the geostrophic wind equation in the atmosphere. 3

(b) What is the concept of front and forntal surface ? Derive the angle between the frontal surface and earth's surface in the atmosphere. 3+5

(c) Drfine virtual temperature and show that if T_v is the virtual temperature, then $T_v = T(1 + 0.61r)$ where r is the mixing ratio, where T be the dry-bulb temperature. 4

9. (a) Derive the adiabetic lapse rate for moist unsaturated air in the atmosphere. 6

(b) What do you mean by adiabatic process ? Deduce the Poisson's equation in the following form

$$\frac{T}{\theta} = \left(\frac{p.}{1000}\right)^{\frac{R}{C_p}}.$$

Where symbols have their usual meanings. 5

(c) Discuss the different cases of pressure changes in the atmosphere with respect to altitude. 5

10. (a) Assuming the sea water is a two component mixture of salt and pure water, show that the principal of conservation of mass leads to the pair of equations

$$\frac{D\rho}{Dt} + \rho div \vec{q} = 0 \text{ and } \rho \frac{Ds}{Dt} = -div \vec{I_s},$$

where symbols have their usual meanings.

(b) Explain the Brunt-Vaisala frequency. Express it in term of c, where symbols have their usual meanings.

8+(5+3)

11. (a) Show that the equation motion of sea water can be expressed as

$$\frac{D\vec{q}}{Dt} = \vec{F} + 2\vec{q} \times \vec{\Omega} - \frac{1}{\rho}\vec{\nabla} p + \frac{\mu}{3\rho}\left[\vec{\nabla}\Theta + 3\nabla^2 \vec{q}\right]$$

(Symbols have their usual meanings.)

(b) Find the condition of stable mechanical equilibrium of stratified sea-water. 12+4

12. Define Salinity and concentration. 2

Or

Define relative and specific humidity.

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