2018

M.Sc.

Part-II Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—VII

Full Marks : 100

Time : 4 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Use separate scripts for each group

Group—A

[Marks : 25]

Answer Q. No. *1* and any *two* from the rest.

1. Define conservative force with an example. 1

2. (a) Prove the following Maxwell's equation :

$$\Delta \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

[Symbols have their usual meanings]

(b) Show that electromagnetic waves are transverse in nature. 8+4

3. (a) Find the relations of electromagnetic fields in terms of electromagnetic potentials.

(b) Deduce Maxwell's equations in terms of electromagnetic potentials. 4+8

4. Define electric image. Find the image of a single point charge 'e' at a distance 'f' from a conducting infinite plane sheet kept at zero potential. Deduce the expression for the surface density of induced charge in this case. 2+4+6

Group—B

(Fuzzy Sets and its application in O.R.)

[Marks : 25]

Answer Q. No. 5 and any *three* from Q. No. 6 to Q. No. 10.

5. Answer any *one* question : 1×2

(a) Define union and intersection of two fuzzy sets.

(b) Determine the α-cut of the trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$.

6. (a) Using subtraction rule for fuzzy numbers, show that $[7,10] - [5,12] = [-5,5]$. 5

(b) Evaluate the following :

$2(5, 7, 9, 13) + 4(-2, 5, 6) - 7[-4, 3] + 6$ 3

7. (a) Let $\tilde{A}$ be the fuzzy set with membership function

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & \text{for } x \le 1 \\ 3(x-1)/8, & \text{for } 1 < x \le 3 \\ (6-x)/4, & \text{for } 3 < x \le 4 \\ (3x-2)/20, & \text{for } 4 < x \le 6 \\ 3(7-x)/5, & \text{for } 6 < x < 7 \\ 0, & \text{for } x \ge 7 \end{cases}$$

Show that it is neither normal nor convex fuzzy set. 4

(b) Give an example to show that the distributive laws donot hold in general for interval numbers. Also, given an example where the distributive laws hold for interval numbers. 4

8. (a) State the Bellman and Zadeh's principle with illustration. 3

(b) Describe Verdegay's method to solve fuzzy LPP. 5

9. (a) Solve the following by Werner's method :

$$\text{Max } z = 4x_1 + 5x_2 + 9x_3 + 11x_4$$

$$\text{Subject to} \quad x_1 + x_2 + x_3 + x_4 \leq \widetilde{15}$$
$$7x_1 + 5x_2 + 3x_3 + 2x_4 \leq \widetilde{80}$$
$$3x_1 + 5x_2 + 10x_3 + 15x_4 \leq \widetilde{100}$$
$$x_1, x_2, x_3, x_4 \geq 0$$

where the tolerances are $p_1 = 5$, $p_2 = 40$, $p_3 = 30$.

(b) Explain the possible ways when the fuzzy environment may occur in a LPP. 6+2

10. (a) Using addition rule of two fuzzy numbers, Prove the addition rule of two interval numbers. 4

(b) Let $\widetilde{A}$ and $\widetilde{B}$ be two fuzzy sets. Prove that

$$(\widetilde{A} \cup \widetilde{B})^C = \widetilde{A}^C \cap \widetilde{B}^C.$$ 2

(c) Define Gaussian fuzzy number with an example. 2

Group—C

[Marks : 30]

11. Answer any *two* questions : 2×3

(a) Write the set of boundary layer equations for 2D steady incompressible flow. 3

(b) Derive the substantial derivative of temperature using chain rule and discuss its physical significance. 3

(c) Define Vontex doublet. 3

12. Answer any *three* questions : 3×8

(a) State and prove the Kutta-Joukowski Lift Theorem. 8

(b) An infinite row of equivalent rectilinear vortices are at a distance *a* apart. The Vortices are the same numerical strength *k* but they are alternately of opposite signs. Find the complex function that determines the velocity potential and the stream function. 8

(c) Let a circular cylinder having radius 'a' be fixed at the origin and x-axis be chosen in the opposite direction of stream U. Find the velocity potential, stream function and hence find the complex potential. 8

(d) Define rectilinear vortex and the find the complex potential due to a vortex of strength k placed at the origin. 2+6

(e) Assuming the necessary stress-strain rate relations, deduce Navier-Stokes equations of motion of an incompressible viscous fluid. 8

Group—D

(Magnetic Hydrodynamics)

[Marks : 20]

Answer any *two* questions : 2×10

13. (a) Write down the basic equations of magneto-hydrodynamics and hence deduce the magnetic induction equation in MHD flows. 2+3

(b) State and prove Alfven's theorem. 1+4

14. (a) Write short notes on "Lorentz force".

(b) Explain, how to work Magneto-Hydrodynamics as a power generator.

(c) Derive the equation of motion of a conducting fluid. 2+2+6

15. (a) For a conducting fluid in a magnetic field, show that the magnetic body force per unit volume, i.e., $\mu(\nabla \times H) \times H$ is equivalent to a tension μH^2 per unit area along the lines of force, together with a hydrostatic pressure $\frac{1}{2}\mu H^2$, where symbols have their usual meaning.

(b) Give the mathematical formulation of MHD couette flow problem. 7+3