C/18/DDE/MSc/Part-I/MTM/5

2018

M.Sc.

Part–I Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—V

Full Marks : 50

Time : 2 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

(*Mechanics of Continuous Media*)

Answer *Q. No. 6* and any *three* questions from the rest.

2+3×16

1. (a) An infinite mass of fluid acted on by a force $\mu r^{-\frac{3}{2}}$ per unit mass directed to the origin. If initially the fluid

is at rest and there is a cavity in the form of the sphere $r = c$ in it, show that the cavity will be filled up after an interval of time $\left(\frac{2}{5\mu}\right)^{1/2} c^{5/4}$. 8

(b) If the equations characterizing the deformation are

$$x_1 = X_1 + \varepsilon X_2$$
$$x_2 = X_2 - \varepsilon X_1 + \varepsilon X_3$$
$$x_3 = X_3 - \varepsilon X_2$$

Determine the Lagrangian and Eulerian finite strain tensors. 8

2. (a) What is the concept of stress vector ? Prove that the stress vector at a point on any arbitrary plane surface is a linear function of three stress vectors acting on any three mutually perpendicular planes through that point. 2+6

(b) Prove that the pressure at a point in a perfect fluid has the same magnitude in every direction. 4

(c) Show that the difference of the values of a two-dimensional stream function at two points represents the flux of a fluid across any curve joining the points. 4

3. (a) The strain tensor at a point is given by

$$(e_{ij}) = \begin{pmatrix} a & b & 0 \\ b & -a & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Find principal strains with its principal direction of strains. 8

(b) Write down the difference between (i) stream line and path line, (ii) source and sink. 4

(c) Define image. Find the image of a source with respect to a straight line. 4

4. (a) Derive the Euler's equation of motion of a perfect fluid. Hence obtain the Bernoulli equation in its most general form.

Bernoulli equation in its most general form. 2+6

(b) Establish the strain vector and strain tensor relationship. 4

(c) Define doublet. Find the complex potential for a doublet. 4

5. (a) Derive the equation of energy for a perfect fluid.

(b) Define strain quadric. Prove that the extension of a line element through the centre of a strain quadric

in the direction of any central radius vector is equal to the inverse of the square of the radius vector. 4

(c) The state of stress throughout a continuum is given with respect to the Cartesian axes $ox_1x_2x_3$ by

$$(T_{ij}) = \begin{pmatrix} 3x_1x_2 & 5x_2^2 & 0 \\ 5x_2^2 & 0 & 2x_3 \\ 0 & 2x_3 & 0 \end{pmatrix}$$

Determine the stress vector acting at the point $(2, 1, \sqrt{3})$ of the plane that is tangent to the cylindrical surface $x_2^2 + x_3^2 = 4$ at P. 4

6. Find the complex potential of a source. 2

Or,

Write down the strain invariants. 2