2018

M.Sc.

Part-I Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—IV

Full Marks : 100

Time : 4 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Write the answer to questions of each group in Separate answer booklet.

Group—A

(*Classical Mechanics*)

[Marks : 50]

Answer *Q. No. 1* and any *three* questions from the rest.

1. Answer any *one* question : 1×2

   (a) What do you mean by generalized coordinates ? How they differ from conventional coordinates ?

   (b) State Hamilton's principle.

2. (a) Consider a mechanical system described by n generalized co-ordinates $q_1, q_2, \ldots, q_n$. Show that the kinetic energy can be formulated as

$$T = \frac{1}{2}\sum_{i,j} a_{ij}\dot{q}_i\dot{q}_j + \sum_i b_i\dot{q}_i + c$$

State the expressions for the coefficients. 4

(b) The potential energy and kinetic energy of a dynamical system are given by

$$V = \frac{1}{2}kr^2 \quad \text{and} \quad T = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\theta}^2 + \frac{1}{2}mr^2\sin^2\phi\dot{\phi}^2$$

Determine the Lagrangian and Lagrange's equations of motion. 4

(c) State Hamilton's principle and derive it from D'Alembert's principle. 8

3. (a) Show that the Coriolis force due to the rotation of earth deflects of vertically falling particle in northern hemisphere toward east and the deflection is proportional to $h^{3/2}$ for a given latitude where h is the height of the fall. 8

(b) In relativistic mechanics, show that the mass of a particle increases with velocity, i.e.,

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$ 5

(c) Show that for a Scleronimic system the Hamiltonian is the total energy of the system. 3

4. (a) Show that the Hamiltonian for the simple oscillator viz,

$$H = \frac{p^2}{2m} + \frac{1}{2}mw^2x^2,$$

can be written in the form $H = wa^*a$, where

$$a = \sqrt{\frac{mw}{2}}\left(x + \frac{ip}{mw}\right) \text{ and } a^* = \sqrt{\frac{mw}{2}}\left(x - \frac{ip}{mw}\right)$$

Evaluate the Poisson brackets $[a, a^*]$, $[a, H]$ and $[a^*, H]$. 8

(b) State and Prove Jacobi's identity for Poisson bracket. 8

5. (a) Prove that $J = \int_{x_0}^{x_1} F(y, y', x)dx$ will be minimum only when

$$\frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) - \frac{\partial F}{\partial y} = 0$$ 6

(b) If the kinetic energy $T = \frac{1}{2}mr^2$ and the potential energy $V = \frac{1}{r}\left(1 + \frac{r^2}{c^2}\right)$, find the Hamiltonian. Determine whether

(i) H = T + V

(ii) $\frac{dH}{dt} = 0$. 8

6. (a) Consider the equilibrium configuration of the molecule such that two of its atoms of each of mass *M* are symmetrically placed on each side of the third atom of mass *m*. All three atoms are collinear. Assume the motion along the line of molecules and there being no interaction between the ends atoms. Compute the kinetic energy and potential energy of the system and discuss the motion of the atoms. 8

(b) Find the equations of motion of a rigid rotating with an angular velocity *w* about a fixed point. 8

Group—B

(Partial Differential Equation)

[Marks : 50]

Answer *Q. No. 1* and any *three* from the rest.

1. State different types of boundary conditions in partial differential equations. 2

Or

Find the characteristics of the equation

$$u_{xx} + 2u_{xy} + \sin^2 x\, u_{yy} + u_y = 0$$

when it is of hyperbolic type. 2

2. (a) Find the general integral of

$$2y(z-3)p + (2x-z)q = y(2x-3)$$

which passes through $z = 0$, $x^2 + y^2 = 2x$. 6

(b) Find the solution of the equation

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y$$ 6

(c) Find the complete integral of the equation $p^2y(1+x^2)=qx^2$. 4

3. (a) Prove that the total energy of a string, which is fixed at the points x = 0, x = L and executing small transverse vibrations, is given by

$$\frac{1}{2}T\int_0^L\left[\left(\frac{\partial y}{\partial x}\right)^2+\frac{1}{c^2}\left(\frac{\partial y}{\partial t}\right)^2\right]dx$$

where $c^2=\frac{T}{P}$, P is the uniform linear density and T is the tension. Show also that if $y=f(x-ct), 0\le x\le L$, then the energy of the wave is equally divided between potential energy and kinetic energy. 5+3

(b) Solve the boundary value problem

$$\Delta^2 u=0, 0\le x\le a, 0\le y\le b$$

with boundary conditions $u(x,b)=u(a,y)=0$, $u=(0,y)=0, u(x,0)=f(x)$. 8

4. Consider the equation :

$$y^2u_{xx}-x^2u_{yy}=0, x>0, y>0$$

What is the nature of the equation in first quadrant? Find the new characteristic coordinates that will change the original equation to canonical form for (x,y) in first quadrant. Find the canonical form of the

above partial differential equation and hence find the general solution u(x,y) of the equation. 2+6+2+6

5. (a) Find the solution of the following partial differential equation using separation of variables method :

$$u_{xx}-u_y+u=0$$ 6

(b) Solve the exterior Dirichlet's problem for a circle given by

$$\nabla^2 u=0, a\le r<\infty, 0\le\theta\le 2\pi$$

$$u(a,\theta)=f(\theta), 0\le\theta\le 2\pi$$

Here '*a*' is the radius of the circle and f(*θ*) is a known function of '*θ*'. 10

6. (a) Establish the Poisson integral formula of the interior Dirichlet's problem for a circle. 8

(b) Show that the Green's function for Dirichlet's problem is symmetric. 4

(c) Show that if *u* solves the Neumann problem. For Poisson's equation, then for any other solution is of the form *v* = *u* + *c* for some real *c*. 4

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