2018

M.Sc.

Part-I Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—III

Full Marks : 100

Time : 4 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Write the answer to questions of each group in Separate answer booklet.

Group—A

(*Probability and Statistics*)

[Marks : 30]

Answer any *two* questions : 2×15

1. (a) Define Markov chain with an example. Define the state and the transition probability. What is transition matrix ? 4

   (b) Prove that $1-r_{1.23}^2=(1-r_{12}^2)(1-r_{13.2}^2)$. Use this relation to show that the multiple correlation coefficient is numerically greater than any of the total or partial correlation coefficients of x_1 with the other variables. 5

(c) Prove that the state j is persistent iff $\sum_{n=0}^{\infty} p_{jj}^{(n)} = \infty$. 6

2. (a) Let $\{X_n, n \geq 0\}$ be branching process. Show that $m = E(X_1) = \sum_{k=0}^{\infty} kp_k$ and $\sigma^2 = \text{var}(X_1)$, then $E(X_n) = m^n$ and $\text{var}(X_n) = \begin{cases} \dfrac{m^{n-1}(m^n - 1)}{m-1}\sigma^2 & , \text{ if } m \neq 1 \\ m\sigma^2 & , \text{ if } m = 1 \end{cases}$ 9

(b) Find the differential equation for Wiener process. 6

3. (a) Find the probability generating function for birth and death process when rate of birth and death are respectively $n\lambda$ and $n\mu$, where n is the population size at any time t. Assume that the initial population size is i. 7

(b) Find the regression equation of X_1 on X_2 and X_3 given the following results :

Trait	*Mean*	*Standard Deviation*	r_{12}	r_{23}	r_{31}
X_1	28.02	4.42	+ 0.80	...	...
X_2	4.91	1.10	...	−0.56	...
X_3	594	85	...	...	−0.40

where X_1 = seed per acre ; X_2 = Rainfall in inches ; X_3 = Accumulated temperature above 42°F. 5

(c) State and prove first entrance theorem. 3

Group—B

(*Numerical Analysis*)

[Marks : 40]

Answer *Q. No.* 4 and any *three* from the rest.

4. Prove the following relations : (any *two*). 2×2

(a) $(1 + \delta^2\mu^2)f(x) = \left(1 + \dfrac{\delta^2}{2}\right)^2 f(x)$

(b) $\Delta \log f(x) = -\log\left[1 - \dfrac{\Delta f(x)}{f(x)}\right]$

(c) $\mu\delta f(x) = \dfrac{\Delta + \nabla}{2} f(x)$

where the symbols have their usual meaning.

5. (a) Deduce Stirling's central difference interpolation formula. State its limitations. 8

(b) Use inverse Lagrange's interpolation to find a root of the equation $x^2 - 3x + 1 = 0$ 4

6. (a) Describe Newton-Raphson method to solve the following non-linear equations $f(x, y) = 0$ and $g(x, y) = 0$.

Using the method, solve the system $x^2-2x-y+0.5=0$ and $x^2+4y^2-4=0$ with the starting values x_0 = 2.0 and y_0 = 0.25. 8

(b) Deduce 3-point Gauss-Chebyshev quadrature formula. 4

7. (a) Describe Milne's predictor-corrector method to solve $\frac{dy}{dx}=f(x,y)$ with $y(x_0)=y_0$. 6

(b) By a suitable numerical method solve the wave equation $\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}, 0 \le x \le 1, t \ge 0$ with initial conditions $u(x,0)=f(x), \left(\frac{\partial u}{\partial t}\right)(x,0)=g(x), 0<x<1$ and boundary conditions $u(0,t)=\varphi(t), u(1,t)=\varphi(x), t \ge 0$. 6

8. (a) Describe the Runge-Kutta method of 4th order to solve the following pair of differential equations $\frac{dy}{dx}=f(x,y,z)$ and $\frac{dz}{dx}=g(x,y,z)$ with initial conditions $y(x_0)=y_0, z(x_0)=z_0$. 4

(b) Suppose $y=1-\frac{x}{2!}+\frac{x^2}{4!}-\frac{x^3}{6!}+\frac{x^4}{8!}-\cdots$

Economize this series if the fourth-decimal places is not to be affected, near x = 1. 4

(c) Find the value of $|A|$ using partial pivoting, where $A=\begin{pmatrix} 0 & 1 & 2 \\ 1 & 5 & 3 \\ -1 & 8 & 6 \end{pmatrix}$. 4

9. (a) Describe LU-decomposition method to solve the system of equation $Ax = b$ with necessary conditions. 10

(b) Explain the need of stability analysis of a numerical method to solve a differential equation. 2

Group—C

(Introduction to Computing)

[Marks : 30]

10. Answer any *six* questions : 6×5

(a) Explain call by value and call by reference. Explain these two in swapping of two values through two programs.

(b) Write a program in C to cheek whether a number is prime or not using a user defined function.

(c) Explain the difference with an example in each between (i) break and continue statements, (ii) while and do-while statements.

(d) What is structure in C ? How does a structure differ from an array ? How can structure variable be declared ? How are the members of a structure variable assigned initial values ? How is a structure member accessed ?

(e) Using Karnaugh map. simplify the following Boolean function: $F(A,B,C,D) = \Sigma(1,3,5,6,8,11,13) + \Sigma_{\phi}(0,2,4)$. 3

(f) Explain the concepts of 'pointer' and 'pointer variable'. How can a pointer variable be initialized? What is the relationship between an array and a pointer ? Illustrate all with examples.

(g) Write a program in C that will read the value of x and evaluate the following function :

$$y(x) = \begin{cases} ae^{-x}, & \text{for } x < 0 \\ \sqrt{x+10}, & \text{for } 0 \leq x \leq 5 \\ \log(x^2+5), & \text{for } x > 5 \end{cases}$$

(h) Design a logic circuit of an overheat alarm for an oil fired steam boiler. In this system, there are three sensors. One of them monitors the water temperature in the boiler, another monitors the chimney temperature and the other follows on-off state (of the burner. An alarm signal should be generated whenever the burner is on and either the chimney or water temperature is too hot. 5

(i) Explain floating point representation of numbers with examples.

(j) Write a program in C to check whether a number is palindrome or not through a user defined function.

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