Total Pages—7 **C/18/DDE/MSc/Part-I/MTM/2**

2018

M.Sc.

Part-I Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—II

Full Marks : 100

Time : 4 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

The symbols have their usual meanings.

Write the answer to questions of each group in Separate answer booklet.

Group—A

(*Algebra*)

[Marks : 50]

Answer *Q. No. 1* and any *three* from the rest.

1. Answer any *one* question : 1×2

(a) Is the converse of Lagrange's Theorem true ? Justify your answer. 2

(b) Define complete graph and complete bipartite graph. 1+1

2. (a) Define partial order set (Poset). Let $D_{48} = \{1, 2, 3, 4, 6, 8, 12, 24, 48\}$ be a set and $(D_{48}, /)$ be a poset, where '/' stands for divisibility. Draw the Hasse diagram of the poset $(D_{48}, /)$. Hence show that $(D_{48}, /)$ is a lattice. 1+4+2

(b) Are the polynomials x^2-7, $x^2 + 9$ irreducible over Q and $\mathbb{R}$? Justify. 2+2

(c) Show that the vertices of every planar graph can be properly coloured with five colours. 5

3. (a) State and prove Sylow's third theorem for a finite graph. 7

(b) State first isomorphism theorem of graph. Show that any epimorphism of Zonto itself is an isomorphism. 2+3

(c) Let G_1 and G_2 be two graphs such that G_1 is isomorphic to G_2. If G_1 is a connected graph, then show that G_2 is a connected graph. 4

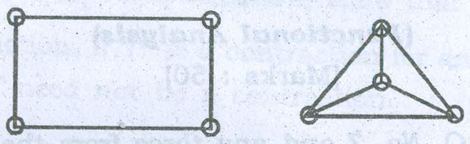
4. (a) Let H be a subgroup of a group G. If $x^2 \in H, \forall x \in G$, then prove that H is a normal subgroup of G and G/H is commutative. 5

(b) Show that the set of matrices $S = \left\{\begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} : a, b \in \mathbb{R}\right\}$ is a left ideal but not a right ideal of all 2×2 real matrices. 4

(c) Show that every circuit has been number of edges common with any edge-cut. 4

(d) Show that the complement of elements of a bounded distributive lattice $(L, \wedge, \vee)$ if exists is unique. 3

5. (a) Define isomorphism between two graphs. Are the following two graphs isomorphic ? Justify. 2+4



(b) Find the characteristic of the ring $(\mathbb{Z}_6, +, .)$ and also find its idempotent elements. 5

(c) If H is a p-Sylow subgroup of G and $x \in G$, then show that $x^{-1}Hx$ is also a p-Sylow subgroup of G. 5

6. (a) (i) Show that no group of order 63 is simple. 2

(ii) Define Boolean ring. Let R be a Boolean ring with identify and I be a proper ideal of R. Show that I is a prime ideal of R if and only if I is a maximal ideal of R. 1+4

(b) Let $(1, \leq)$ be a distributive lattice with 1 and 0. Show that

(i) every element of L has atmost one complement. 2

(ii) $(a \wedge b) \vee (b \wedge c) \vee (c \wedge a) = (a \vee b) \wedge (b \vee c) \wedge (c \vee a)$ for all $a, b, c \in L$.

(c) (i) Does there exist a simple connected planar graph with 5 vertices and 100 edges ? Justify. 2

(ii) Define $\chi(G)$ of a graph G. Find $\chi(K_n)$ for the graph K_n. 2

Group—B

(Functional Analysis)

[Marks : 50]

Answer Q. No. 7 and any *three* from the rest.

7. Answer any *one* question : 2×1

(a) Define contraction mapping with an example.

(b) Give an example of a linear operator which is not bounded.

8. (a) Establish a necessary and sufficient condition for a function $f:(X,d_1)\to(Y,d_2)$ to be continuous. 4

(b) Show that every compact metric space is separable. 4

(c) Prove that a metric space (x, d) is not of first category if x is complete. 8

9. (a) Use Banach Fixed Point theorem, to show that the differential equation

$$\frac{dy}{dx}=\varphi(x,y)$$

has a unique solution passing through the point $(x_0,y_0)\in D$, where $\varphi(x,y)$ and $\frac{\partial\varphi}{\partial y}$ are continuous on the closed rectangle D. 8

(b) If $T:X\to X$ is a contraction, show that $T^n (n\in\mathbb{N})$ is a contraction. If T^n is a contraction for an $n>1$, show that T need not be a contraction. 4

(c) Let $T:\mathbb{R}\to\mathbb{R}$ be defined by $T_x=2(1-\frac{x}{5})$. Use Banach Fixed Point theorem to find the fixed point of T as a limit of the iterative sequence. 4

10. (a) Let $T:V\to W$ be a linear transformation where V and W are normal linear spaces. Show that the following are equivalent— 5

(i) T is continuous at $0\in V$.

(ii) T is continuous at all points of V.

(iii) T is bounded.

(b) Let V be a non-zero normed space. Is $V^* \neq \{0\}$? Justify your answer. 5

(c) Let X and Y be Banach spaces and $T : X \to Y$ is a linear map which has a closed graph. Show that $T \in BL(X,Y)$. 6

11. (a) If T is a bounded linear function on a normed space X and $\{x_n\}$ is Cauchy in X, then show that $\{Tx_n\}_{n\geq 1}$ is a Cauchy sequence. 3

(b) Show that every inner product space is a normal space but not conversely. 5

(c) Let H be a Hilbert space and F be a closed subspace of H. Show that for every $x \in H$, there exist a unique $y \in F$ and $z \in F^{\perp}$ such that $x = y + z$. 8

12. (a) Let H be a Hilbert space and $y \in H$. Let $\phi_y : H \to C$ be defined by $\phi_y(x) = < x, y >, \forall x \in H$. Show that $\phi_y \in H^*$ and $\|\phi_y\| = \|y\|$. Also, show that if $\phi \in H^*$, then there exist unique $y \in H$ such that $\phi = \phi_y$. 7

(b) Let $T_1, T_2 \in BL(H)$ be normal operators such that $T_1T_1^* = T_1^*T_1$ and $T_2T_2^* = T_2^*T_2$. Show that $(T_1 + T_2)$ and T_1T_2 are also normal. 5

(c) Let H be a Hilbert space and FCH. Define $F^{\perp}$ and show that it is a closed linear subspace of H. 4

———