2018

M.Sc.

Part-II Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—X (OR/OM)

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Special Paper : OR

Full Marks : 100

Time : 4 Hours

Answer Q. No. *11* and any *six* from the rest.

[Calculator may be used]

1. (a) Obtain the probability P_n of n customer in the system for the $(M/M/C):(\infty/FCFS/\infty)$ queuing model.

(b) Describe different type of service discipline. Define the following terms :

mean servicing rate, mean arrival rate and traffic intensity. 10+6

2. (a) Define entropy and show that it is continuous. 4

(b) Define joint and conditional entropies. Prove that $H(X,Y) \le H(X) + H(Y)$ with equality if X and Y are independent. 8

(c) Write an easay on information theory emphasizing the basic concepts. 4

3. (a) What is simulation? Describe its advantages in solving the problems Give its main limitations. 4

(b) Explain Monte-Carlo simulation. State different mathematical steps in Monte-Carlo method. 4

(c) Describe a method to generate random numbers. Use your method to generate first 5 random numbers. 5

(d) Describe a simulation based method to find the value of π. 3

4. (a) (i) What do you mean by network analysis? What are the advantages of it? 3

(ii) Explain a method to numbering the nodes in the network. 3

(iii) How do you calculate the earliest starting time and the earliest finish time? 2

(b) The following are the details of estimated times of activities of a certain project :

Activity :	A	B	C	D	E	F
Immediate predecessor :	—	A	A	B,C	–	E
Estimated time (weeks) :	2	3	4	6	2	8

(i) Draw the network.

(ii) Find the critical path and the expected time of the project.

(iii) Calculate the float for each activity. 8

5. (a) A man is engaged in buying and selling identical items. He operates from warehouse that can hold 500 items. Each month he can sell any quantity that he chooses upto the stock at the beginning of the month. Each month, he can buy as much as he wishes for delivery at the end of the month so long as his stock does exceed 500 its. For the next four month, he has the following error-free forecasts of cost sales prices:

Month :	(i)	4	3	2	1
Cost :	(c_i)	27	24	26	28
Sale price :	(p_i)	28	25	25	27

If the current stock is 200 units, What quantities should he sell and buy in next four months? Find the solution using dynamic programming method. 8

(b) Solve the following LPP using dynamic programming method

Maximize $z = 8x_1 + 7x_2$

Subject to $2x_1 + x_2 \le 8$

$5x_1 + 2x_2 \le 15$

$x_1, x_2 \ge 0$. 8

6. (a) Find the optimal control u and the optimal path that extremizes

$$J = \frac{1}{2}\int_0^{\infty}\left(x^2 + 4u^2\right)dt$$

Where $\ddot{x} = -\dot{x} + u$, and $u(0) = \alpha$, $\dot{x}(0) = \beta$ both x and $\dot{x} \to 0$ as $t \to \infty$. 8

(b) Define the reliability of a system. What are MTBF and MTTF? Show that :

$R(t) = \exp\left[-\int_0^t \lambda(t)dt\right]$, where R(t) is the reliability function and $\lambda(t)$ represents the failure rate. 8

7. (a) In a certain manufacturing situation, the production is instantaneous and the uniform rate of demand is D. Show that the optimal order quantity is

$$Q = \sqrt{\frac{2C_3(C_1 + C_2)}{C_1 C_2}}$$

Where C_1, C_2 are the holding and shortage cost per unit time and C_3 is the setup cost per order. 8

(b) The annual requirement for a product is 3000 units. The ordering cost is Rs. 100/- per order. The cost per unit is Rs. 10/-. The carrying cost per year is 30% of the unit cost.

(i) Find the EOQ.

(ii) If a new EOQ is found by using the ordering cost as Rs. 80/-, What would be further saving in cost? 8

8. (a) Find the optimal sequence for following sequencing problem of five jobs and three machines, when passing is not answered. Its processing time (in hours) is given below. 8

Job :	1	2	3	4	5
Machine A :	8	10	6	7	11
Machine B :	5	6	2	3	4
Machine C :	4	9	8	6	5

(b) What is replacement? Describe the optimal replacement policy (s) for items whose running cost increases with time in discerete units and the value of money remains constant during a period. 8

9. (a) Minimize the following function by geometric programming.

$$f(x) = x_1 x_2 x_3^{-2} + 2x_1^{-1} x_2^{-1} x_3 + 5x_2 + 3x_1 x_2^{-2}$$

$x_1,\ x_2,\ x_3 > 0$. 8

(b) Define a non-cooperative game with n players. What is equilibrium situation? When two games are said to be strategically equivalent? Show that strategically equivalent games obey symmetric and transitive properties. 8

10. (a) Find the optimum order quantity for a product for which the price breaks are as follows :

Quantity	*Purchase Cost (per unit)*
$0 \le Q_1 < 100$	*Rs.*20
$100 \le Q_2 < 200$	*Rs.*18
$200 \le Q_3$	*Rs.*15

The monthly demand for the product is 400 units. The shortage cost is 20% of unit cost of the product and the cost of ordering is Rs. 25 per month. 8

(b) At time zero, all items in a certain system are new. Each item has a probability p of falling immediately before the end of the first month of life and a probability 1 – p (=q) of failing immediately before the end of the second month. If all items are replaced as they fail. Show that the expected number of failures f(x) at the end of the month x is $f(x) = \frac{N}{1+q}\left[1-(-q)^{x+1}\right]$

N being the number of items in the system. Hence find the number of replacement under the individual policy. 8

11. Answer any one of the following : 1×4

(a) Explain the terms 'shortage' and 'lead' time' in connection with inventory.

(b) What is continuous game? State the fundamental theorem for continuous game.

Special Paper : OM

Full Marks : 75

Time : 3 Hours

1. Answer any *five* questions : 2×2

(a) Derive the equivalence property of Tephigram and discuss its different properties. 7

(b) Deduce the equation of state for moist air in the following form

$$p\alpha = \frac{R^*}{m_d}\left(\frac{1+\frac{w}{\varepsilon}}{1+w}\right)T$$

Where symbols have their usual meanings. 5

(c) Show that for an isentropic process, $p\alpha^{\gamma} = \text{constant}$. 3

2. (a) Find the rate of change of circulation in the atmosphere and interpret each term. 7

(b) Define specific entropy and establish the relationship between the specific entropy and the potential temperature. 5

(c) Define virtual temperature and show that if T_v is the virtual temperature, then $T_v = T(1+0.61r)$ where r is the mixing ratio. 3

3. (a) define homogeneous atmosphere. Show that the height of the homogeneous atmosphere depends entirely on the temperature at the bottom. Also prove that the pressure at the top of the homogeneous isothermal atmosphere is equal to $\frac{1}{e}$ times that at the sea level. 7

(b) Derive the pressure tendency below a frontal surface in the atmosphere. 5

(c) Express the first law of thermodynamics in the form $dq = C_p dT - \alpha dp$, where symbols have their usual meanings. 3

4. (a) How is the thermal wind formed in the atmosphere? Derive the thermal wind components in the atmosphere. 7

(b) Derive a relation from which the saturation temperature can be obtained when the saturation of the unsaturated moist air will be done by adiabatic ascent. Hence estimate the height at which saturation is attend. 8

5. (a) Derive the momentum equation of an air parcel in the atmosphere. 7

(b) Derive the expression of water vapor content in an air column in the atmosphere. 5

(c) What is Rossby Wave ? Find the variation of coriolis parameter to create Rossby Wave. 3

6. (a) Derive the effect of ascent and descent of an air parcel on lapse rate in terms of pressure changes. 7

(b) Discuss the different cases of pressure changes in the atmosphere with respect to altitude. 5

(c) Derive the pressure tendency equation. 3

7. (a) What is turbulent motion ? Explain turbulent transfer of momentum in the atmosphere. 7

(b) Derive the expression of the pressure gradient force in the atmosphere. 4

(c) Derive the adiabatic lapse rate of unsaturated moist air. 4

8. (a) State and prove the Clausius-Clapeyron equation in the atmosphere. 7

(b) Derive the hypsometric equation in the atmosphere. 4

(c) What do you mean by adiabatic process ? Show that the adiabatic process is more steeper than he isothermal process. 4