2018

BCA 1st Semester Examination

DISCRETE MATHEMATICS

PAPER—1103

Full Marks : 70

Time : 3 Hours

The figures in right-hand the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Answer Q. No. *1* and any *six* questions from the rest.

1. Answer any *five* questions : 5×2

(a) If A = {2, 5}, B = {3, 5, 7}, find (A × B) ∩ (B × A).

(b) Define a group.

(c) Define a complete graph. Draw K_4.

(d) How many vertius are there in a 4-regular graph with ten edges ?

(e) If a group (G, o) is abelion, then show that $(ab)^2 = a^2b^2$, for all a, b in G.

(f) If $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 1 & 5 & 3 \end{pmatrix}$ then find inverse per mutation f^{-1}.

(g) Construct a truth table for $(\sim p \wedge q) \vee p$.

(h) Find the recurrence relation of the sequence

{1, 1, 2, 3, 5, 8, 13, 21, ...}.

2. (a) Use mathematical induction to prove that $8^n - 3^n$ is a multiple of 5 for $n \geq 1$. 5

(b) Solve the recurrence relation $a_n = a_{n-1} + n, n \geq 1$ subject to initial condition $a_n = 1$, for $n = 0$. 5

3. (a) Let S be the set of all positive division of 72. Define a relation ≤ on s by "x ≤ y if and only if x is a divisor by y" for x, y ∈ s. Prove that (S, ≤) is a poset. Draw the Hasse diagram of the poset. 3+3

(b) Examine whether the mapping f : Z → Z defined by f(x) = 2x + 1 for all x ∈ z is injective or surjective or both. 4

4. (a) Show that $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$ is a tautology. 5

(b) A relation ρ on the set of all integers Z is defined as $\rho = \{(a, b) \in z \times z : 2a + 3b$ is divisible by $5\}$. Examine whether ρ is an equivalence relation or not. 5

5. (a) Draw a graph with the help of adjacency matrix

$$\begin{pmatrix} 1 & 1 & 1 & 2 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 2 \\ 2 & 0 & 2 & 2 \end{pmatrix}.$$ 5

(b) Prove that the set $\{z^n : n \in z\}$ forms a commutative group with respect to multiplication. 5

6. (a) Prove that the roots of $x^4 = 1$ forms a cyclic group under multiplication. Find its generators. 5

(b) Prove that a field is an integral domain. 5

7. (a) Find the number of possible ways in which the letters of the word 'ESSAY' can be arranged so that the two 's' don't come together. 5

(b) Simplifying the Boolean expression using k-map

$$A\overline{BC} + AB\overline{C} + ABC + A\overline{B}C + \overline{A}B\overline{C} \cdot$$ 5

8. (a) Show that maximum number of edges in a simple graph with n vertices is $n(n-1)/2$. 5

(b) Define an Eulerian circuit. Give an example. 3

(c) Define a planar graph with an example. 2

9. (a) Prove that every connected graph has at least one spanning tree. 5

(b) Prove that the sum of the degrees of all the vertices in any graph G is twice the number of edges in G. 5

10. (a) Obtain DNF for $\sim(p \rightarrow (q \wedge r))$. 4

(b) Draw graphs for the followings :

(i) 4-regular graph on 6 vertices,

(ii) Eulerian circuit but not Hamiltonian circuit,

(iii) Spanning tree of K_2, 3. 6