UG/III/PHS/H/VI/18(New)

2018

PHYSICS

[Honours]

PAPER – VI

Full Marks : 90

Time : 4 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

[NEW SYLLABUS]

GROUP – A

Answer any two of the following questions : 15×2

1. (*a*) A particle of mass m and total energy E is incident on a rectangular potential barrier of width δ and height $V_0 > E$, where

$$V_0(x) = V_0 \text{ for } 0 < x < \delta$$
$$= 0 \quad \text{for } x < 0 \text{ and } x > \delta.$$

Derive the transmission co-efficient for broad and high barrier. 8

(*b*) The quantum mechanical transmission coefficient of an α-particle through a nuclear potential barrier is 2×10^{-24}. Taking the velocity of the α-particle and the nuclear radius as 10^7 m/s and 10^{-14} m respectively, calculate the mean life time of α-decay. 2

(*c*) Consider the operator

$$\hat{Q} \equiv \frac{d^2}{d\phi^2},$$

where ϕ is the azimuthal angle in polar coordinates. Is $\hat{Q}$ hermition ? Find its eigen functions and the eigenvalues. Is the spectrum degenerate ? 2 + 2 + 1

2. (*a*) What are the basic interactions in nature ? Give the relative strength, Range and mediators of these interactions. 2 + 3

(*b*) What is meant by octet symmetry ? Demonstrate the octet symmetry of baryons in a weight diagram. 2 + 3

(*c*) Write the composition of proton in terms of quarks and hence find the charge, the baryon number, strangeness and the spin of proton. 1 + 2

(*d*) Check if the following reaction is allowed or not in terms of charge, baryon number, strangeness and hypercharge.

$$p + \bar{p} \rightarrow 2\pi^{+} + 2\pi^{-} + 2\pi^{\circ}$$ 2

3. (*a*) What are symmetric and antisymmetric wave functions ? Show how antisymmetric wave function Obey's Pauli's exclusion principle. Construct symmetric and antisymmetric wave functions for three spinless identical particles from unsymmetrised wave function. (1 + 1) + 2 + (2 + 2)

(*b*) Assuming angular momenta commutation relations i.e. $\left[\hat{L}_x, \hat{L}_y\right] = i\hbar\hat{L}_z$ etc. and $\hat{L}_{\pm} = (\hat{L}_x \pm i\hat{L}_y)$, show that

(*i*) $\left[\hat{L}^2, \hat{L}_\pm\right] = 0$ and $\left[\hat{L}_z, \hat{L}_\pm\right] = \pm\hbar\hat{L}_\pm$,

(*ii*) $\hat{L}_z(\hat{L}_\pm Y_{l,m}) = (m \pm 1)\hbar Y_{l,m}$

(*iii*) $\left\langle \hat{L}_x \right\rangle = 0 = \left\langle \hat{L}_y \right\rangle$, while $\left\langle \hat{L}_x^2 + \hat{L}_y^2 \right\rangle \neq 0$.

Note that the expectation value results state $Y_{l,m}$.

[All the symbols used above are of usual meaning. A proof either for $\hat{L}_+$ or for $\hat{L}_-$ is sufficient.]

3 + 2 + 2

4. (*a*) Find out the chemical potential of a very dilute gas consisting N structure less particles occupying a volume V at temperature T.

[Given that $\int_0^\infty e^{-\alpha x^2} x^2 dx = \frac{\sqrt{\pi}}{4}\alpha^{-3/2}$;

α is a constant]

Indicate the condition of validity of the expression of chemical potential at classical limit. At what condition it can be less than KT? K is the Boltzmann constant. 4 + 1 + 1

(*b*) For one-fermion in a given particle-state, write down the partition function for the j-th particle state. Hence, find the mean occupation number of this state. How does this differ for Boson ? Indicating the limiting condition find its expression for the particle following M-B statistics. 3 + 1 + 1

(*c*) Derive the equation of state of weakly degenerate fermion-gase.

$$\left[\text{Given that } \int_0^\infty e^{-\beta}\beta^{3/2}d\beta = \frac{3}{4}\sqrt{\pi}\right] \quad 4$$

GROUP – B

Answer any **five** questions : 8 × 5

5. (*a*) What are meant by latitude and altitude effects in relation to cosmic rays ? How are these two effects explained ? 3 + 3

(*b*) A pion decays from rest to give a muon of energy 3.57 MeV to the following scheme

$$\pi^+ \rightarrow \mu^+ + \gamma + Q.$$

Find the energy of the associated neutrino.
Given : $m(\pi) = 273\ m_e$, $m(\mu) = 207\ m_e$
$m_e = 0.51$ MeV. 2

6. (*a*) Consider a particle executing simple harmonic motion is assumed to be confined to a region $\sim a$. Using the uncertainty principle, obtain the minimum energy of the particle. 3

(*b*) At time $t = 0$, the wavefunction for the hydrogen atom is given by

$$\psi(0) = \frac{1}{\sqrt{10}}\left[\psi_{210} + \sqrt{2}\psi_{211} + \sqrt{3}\psi_{21-1}\right]$$

where the subscripts are the values of the quantum no. n, l, m and ψ_{nlm} is the eigen state.

(*i*) Find the wave function at time t.

(*ii*) What is the expectation value for the energy of the system ?

(*iii*) What is the probability of finding the system with ψ_{211} ? 1 + 1 + 1

(*c*) Prove that

$$L_- L_+ = L^2 - L_z^2 - \hbar L_z,$$

symbols have their usual meanings. 2

7. (*a*) A solid containing noninteracting paramagnetic atoms, each having a magnetic moment equal to one Bohr magneton is placed in a magnetic field to strength 3 tesla. Assuming that the atoms are in thermal equilibrium with the lattice, find the temperature to which the solid must be cooled, so that more than 60% of the atoms are polarised parallel to the magnetic field.

(Given : $\mu_B = 9 \cdot 27 \times 10^{-24} J/T$) 3

(*b*) From the partition function derive the law of equipartition of energy for a system of ideal gas particles. 3

(*c*) Determine the phase trajectory of a bullet of unit mass fired straight upwards with an initial speed of 40 m/s. Acceleration due to gravity is 10 m/s^2. 2

8. (*a*) Deduce Bragg's law in reciprocal lattice space. 4

(*b*) What are Brillouin zones ? Determine K values of the first Brillouin zones for a simple square lattice of side *a*. 1 + 3

9. (*a*) Determine whether electron gas in copper at room temperature is degenerate or nondegenerate. (Concentration of electrons is $(8{\cdot}5 \times 10^{28})$. 2

(*b*) Prove that for a system at T > 0K obeying FD statistics, the probability that a level lying ΔE below the fermi-level is unoccupied is the same as the probability of occupation of a level lying ΔE above the fermi level. 3

(*c*) The dispersion relation of spin $\frac{1}{2}$ fermions in two dimension is $E = \hbar v |\vec{K}|$, where E is the energy, $\vec{K}$ is the wave vector and v is the constant velocity. Find the no. of particles

per unit area if the fermi energy at zero temp is ϵ_F. 3

10. (*a*) The energy of a free electron in a crystal is given by $E = A - B\cos Ka$, where A and B are const., K is the wave number, and a is the distance between adjacent atoms. Draw *E-K* curve and find the effective mass at the top and bottom of the band. If $a = 2$ Å, what is the range of the deBroglie wave length in the 1st Brillouin zone. 1 + 2 + 1

(*b*) Show that the effective potential energy of an electron in the hydrogen atom is minimum at $r = l(l+1)a$. Also show that the minimum effective potential energy is negative. a is the first Bohr radius and l is the orbital quantum no. 2 + 2

11. (*a*) What do you mean by equal-a-priori probability ? Distinguish between canonical and grand canonical ensumbles ? 2 + 2

(*b*) In one dimensional box of length L, a particle with constant velocity is mirror

-reflected at the ends. Draw its phase trajectory. Calculate the phase space volume $\Gamma(E)$ with energy less than E. 2 + 2

12. (*a*) If an operator $\hat{p}$ is Hermition, prove that the operator $\hat{q} = i\hat{p}$ is anti-Hermition. 2

(*b*) Why is an observable in quantum mechanics represented by Hermition operator ? Examine whether $\frac{d}{dx}$ is Hermition or not. 1 + 2

(*c*) Extablish the commutation relation $\left[\hat{x},\left[\hat{x},\hat{H}\right]\right] = \hbar^2/m$, where $\hat{H}$ is the Hamiltonian operator $\hat{H} = \frac{\hat{p}^2}{2m} + \hat{v}(x)$ with the potential $v(x)$ and m as the mass of the particle. 3

GROUP – C

Answer any five questions : 4 × 5

13. (*a*) Determine the value of packing fraction for fcc structure. 2

(*b*) A plane makes intercepts of 1, 2 and ∞ Å on the crystallographic axes of an orthorhombic crystal with $a : b : c = 2 : 1 : 1$. Determine the miller indices of this plane. 2

14. (*a*) Write the partition function of a harmonic oscillator whose energy levels are quantised. 1

(*b*) Find the average energy of the vibration of this oscillator from this partition function. 1

(*c*) According to Einstein model for lattice specitic heat, find the mean energy of the crystal. 1

(*d*) Determine specific heat of the crystal. 1

15. (*a*) What is a four factor formula in a nuclear reactor ? Describe on what each of the factors depend. 2

(*b*) An atomic bomb consisting of ^{235}U explodes and releases an energy of 10^{14} J. Each ^{235}U undergoes fission and release 3 neutrons and about 200 MeV of energy. Further only 20% of ^{235}U atoms in the bomb undergo fission. Find the total number of released neutrons. 2

16. (*a*) Partition function of a gas of N molecules in a volume V and internal energy U is given by

$$z = (V - Nb)^N \left(\frac{aV}{N}\right)^{3N/2},$$

Where a and b are constant. Find the equation of state of the gas, if there is no intermolecular attraction. 2

(*b*) At the same temperature, which and why will exert the greatest and the least pressure-a gas obeying MB statistics, a gas of bossons and a gas of fermions. 2

17. (*a*) Find the frequency of revolution of the ion in a cyclotron. 2

(*b*) The maximum energy of deuterons coming out of a cyclotron is 200 MeV. Calculate the maximum energy of protons that can be obtained from accelerator. 2

18. Show that if the partition function is given by Z, the mean energy $< E >$ is given by

$$\langle E \rangle = -\frac{\partial}{\partial \beta}(\log_e Z), \text{ where } \beta = 1/kT \qquad 4$$

19. Briefly discuss the effect of magnetic field on a superconductor. What is persistent current? Distinguish between type-I and type-II super conductors. $2 + 1 + 1$

20. Show that the reflection coefficient R for a step barrier with condition that the total energy $E > V_0$, V_0 is the height of the barrier, is given by the expression.

$$R = \left(\frac{1 - \sqrt{1 - V_0/E}}{1 + \sqrt{1 - V_0/E}} \right)^2 . \qquad 4$$