Total Pages—15

UG/I/PHS/H/II/18

2018

PHYSICS

[Honours]

PAPER – II

Full Marks : 90

Time : 4 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

GROUP–A

Answer any **two** questions : 15 × 2

1. (*a*) A rain drop of mass m falls from rest at a place where the air resistance is proportional to the velocity v and is kv, where k is a real positive constant.

(*Turn Over*)

(*i*) Set up the equation of motion.

(*ii*) Derive the expression for the velocity of the drop as a function of time. Draw *v*-*t* curve.

(*iii*) Show that the terminal velocity of the drop is $v_T = \frac{mg}{k}$. 4

(*b*) Three particles of masses 2, 3 and 4 units move under the influence of a force field so that their position vectors relative to the fixed coordinate system are given respectively by

$$\vec{r}_1 = 2t\hat{x} - 3\hat{y} + t^2\hat{z},\ \vec{r}_2 = (t+1)\hat{x} + 3t\hat{y} - 4\hat{z},$$
$$\vec{r}_3 = t^2\hat{x} - t\hat{y} + (2t-1)\hat{z},$$

where *t* is the time. Find (*i*) the total angular momentum of the system and (*ii*) the total external torque to the system, taken with respect to the origin. 4

(*c*) Determine the moment of inertia of a cone, of mass M, radius R and height h, about its generating line (i.e. a straight line on its sloping face) 4

(*d*) Determine the ellipsoid of inertia of the cone at its vertex. Find the relation between R and h, so that the ellipsoid of inertia becomes a sphere. 3

2. (*a*) The probability of a gas molecule having velocity component laying between u and $u + du$ in a definite direction at temperature T is given by

$$f(u)du = ae^{-bu^2}\, du$$

where a and b are constants.
Use above formula to show the number of molecules having speed lying between speed c and $c + dc$ is given by

$$N(c)dc = 4\pi N\left(\frac{m}{2\pi k_B T}\right)^{3/2} e^{-mc^2/2k_BT}\, c^2 dc$$

where N is the total number of identical molecules in a vessel at temperature T. Show it graphically (*i*) for two different temperatures T_1 and T_2 ($T_1 > T_2$) and (*ii*) for two different molecules, say H_2 and O_2. Find also the expression for most probable speed. 3 + 2 + 1

(*b*) Show that the fraction of gas molecules whose speeds differ by less than 1% from the value of the most probable speed (i.e. $c_m - 0{\cdot}01c_m < c < c_m + 0{\cdot}01c_m$ or $c \sim c_m < 0{\cdot}01\, c_m$) is about 1.66%. 3

(*c*) What do you mean by transport phenomena ? Given that the net transport of a physical entity H (varying along Z direction) of gas molecules to unit area of a reference plane per unit time is $\frac{1}{3} n\bar{c}\,\lambda \frac{dH}{dz}$ where the symbols are usual. Taking proper choice of H obtain the relation between thermal conductivity K and coefficient of viscosity η of a gas : $K = \eta\, C_V$, where C_V is the specific heat at constant volume. State how far the relation agrees with the experimental result and suggest the possible explanation of the same. 1 + 3 + 2

3. (*a*) How do you distinguish between reversible and irreversible process ? 2

(*b*) Show that the entropy of the universe always increase in an irreversible process. 4

(*c*) Two identical monatomic perfect gases with the same pressure and same number of molecules N are kept in two bulbs of volumes V_1 and V_2 at temperatures T_1 and T_2 respectively. If two bulbs are now connected and the gases mixed, show that the change in entropy after attaining equilibrium is given by

$$\Delta S = 5NK_B \ln\left[(T_1 + T_2)/2\sqrt{T_1T_2}\right]$$

Interpret the positive value of ΔS. 4 + 1

(*d*) A hollow sphere of inner and outer radii 2cm and 3cm is heated by passing current through a tiny heater placed at its centre. In the steady state the inner and outer surface temperatures are 50°C and 30°C respectively. Calculate the power of the heater. [Thermal conductivity of the material of the sphere = $201{\cdot}3\,Jm^{-1}s^{-1}{}^{\circ}C^{-1}$]. 4

4. (*a*) A point charge q is placed in front of an infinite earthed conducting plane. By the method of electrical image find the surface density of induced charge on the conducting plane. 4

(*b*) Determine the magnetic vector potential at a distance ρ from a very long thin straight conductor carrying current I. Hence find the corresponding magnetic field $\vec{B}$. 4

(*c*) A resistance of 20 Ohm and inductance of 200 mH and a capacitance of 100 μF are connected in series across 220V, 50 Hz. Determine the current, power factor and also the power consumed. Draw the phasor diagram. 5

(*d*) Obtain the dimensions of $\vec{B}$ and μ_0. 1 +1

GROUP—B

Answer any five questions : 8 × 5

5. (*a*) Writing u = 1/r prove that the total energy of a particle acted upon by a central force is given by

$$E = \frac{ml^2}{2}\left[u^2 + \left(\frac{du}{d\theta}\right)^2\right] + V(r)$$

where l is the angular momentum per unit mass and $V(r)$ is the potential energy of particle of mass 'm'. Differentiating above relation obtain the equation of motion.

$$\frac{d^2u}{d\theta^2} + u = -\frac{f\left(\frac{1}{u}\right)}{ml^2u^2}.$$ 3 + 2

(*b*) The orbit equation of a particle in central force field is given by $r = a(1 + \cos\theta)$. Show that the force field is attractive and $f(r) = -\frac{k}{r^4}$, where $k > 0$. 3

6. (*a*) Consider a vector $\vec{A}(t)$ in cylindrical co-ordinates (ρ, φ, z). Show that

$$\frac{dA}{dt}=\left(\frac{d\vec{A}_\rho}{dt}-A_\varphi\dot{\varphi}\right)\hat{\rho}+\left(\frac{dA_\varphi}{dt}+A_\rho\dot{\varphi}\right)\hat{\varphi}+\frac{dA_z}{dt}\hat{Z}$$

If $\vec{A}$ in the above expression is the position vector of a particle, determine the corresponding expression of $\frac{d\vec{A}}{dt}$ i.e. the velocity of the particle. Hence by proper choice of $\vec{A}$ determine the acceleration of the particle. 5

(*b*) Show that the kinetic energy of a rigid body rotating about a fixed point (say *O*) is given by :

$$T=\frac{1}{2}\vec{\omega}\cdot\vec{L}.$$

where $\vec{\omega}$ is the angular velocity of the rigid body and $\vec{L}$ is the angular momentum of the rigid body with respect to the point *O*. 3

7. (*a*) What is Brownian motion ? Derive Einstein's equation for mean square displacement of Brownian particle. 1 + 5

(*b*) Show by equipartition of energy that the motions of relatively large Brownian particles are practically unnoticeable. 2

8. (*a*) What is adiabatic lapse rate ? Derive the necessary formula and estimate the adiabatic lapse rate for a dry atmospheric air. Given for dry air : $\gamma = 1{\cdot}4$, molecular weight $= 0{\cdot}029$ kg mol^{-1} and

$R = 8{\cdot}31$ J mol^{-1} K^{-1}. 5

(*b*) If the state functions x, y, z are related by an equation of state $f(x, y, z) = 0$, then show that :

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1.$$

Verify this relation for P, V and T of an ideal gas. 3

9. (*a*) State and prove Carnot's theorem relating to efficiency of engine. 4

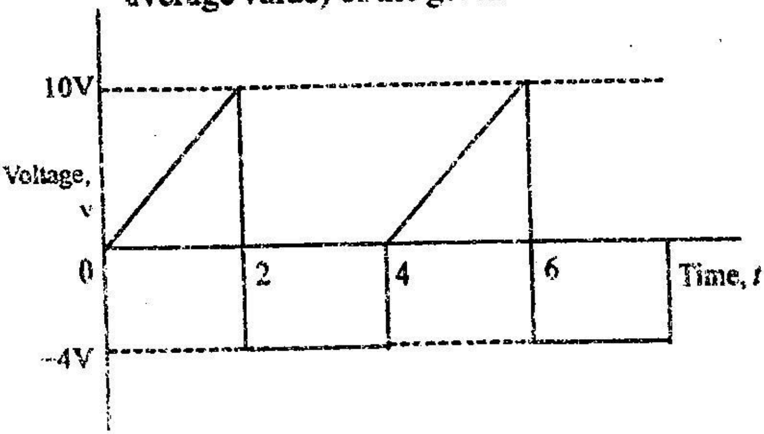
(*b*) Prove : 2 + 1 + 1

(*i*) $\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P$

(*ii*) $\left(\frac{\partial U}{\partial V}\right)_T = 0$ for ideal gas

(*iii*) $\left(\frac{\partial U}{\partial V}\right)_T = \frac{a}{V^2}$ for real gas.

10. (*a*) Find the form factor (i.e. the ratio of rms to average value) of the given waveform : 4



(*b*) Obtain the expression of the electric field due to an electric dipole of dipole moment $\vec{p}$ at position $\vec{r}$ with respect to the centre of the dipole, where r is large compared to the length of the dipole. 4

11. (*a*) Show that for a non-uniform magnetization $\vec{M}$ is equivalent to a bound current density $\vec{J}_b = \vec{\nabla} \times \vec{M}$ through the magnetized object. 4

(*b*) Starting from Ampere's law $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ obtain the modified form of it for a magnetized material $\oint \vec{H} \cdot d\vec{l} = I_f$, where the symbols are usual. 4

12. (*a*) A cylindrical wire of permeability μ carries a steady current I of uniform density. If the radius of the wire be R, find $\vec{B}$ and $\vec{H}$ outside and inside of the wire. 4

(*b*) Find the ampere-turns required to produce a flux of 5×10^{-4} Wb in a ring of iron of 20 cm mean diameter and cross-section 4 cm^2 cut into two equal halves and separated 0·05 cm gap of air on each side. Given the relative permeability of iron = 1250. Neglect the fringing effect and flux leakage. 4

GROUP–C

Answer any five questions : 4×5

13. Consider two concentric thin spherical shells of uniform surface density of masses M_1 and M_2 and Radii R_1 and R_2 $(R_1 > R_2)$ respectively. Find the gravitational force on a particle of mass m when it is placed at a distance r from the centre and (*i*) $r > R_2$, (*ii*) $R_2 > r > R_1$. 4

14. A bullet weighing 125 g, is fired from a rifle at a velocity 800 m/s horizontally towards east at a place of latitude 45°N. Calculate the directions and magnitudes of the horizontal and vertical components of the Coriolis force on the bullet. 4

15. It is given that, for a monochromatic which obeys van der Waals equation, the molar kinetic energy

$U = \frac{3}{2}RT - \frac{a}{V}$ where 'a' is a constant and the other symbols have the usual meaning. The gas initially with volume V_1 and temperature T_1 is allowed to expand adiabatically so that the final volume is V_2. What is the final temperature of the gas ? What would have been the final temperature if the gas were a perfect gas ? 4

16. Derive the relation between the ratio of specific heats of an ideal gas and the degrees of freedom of a molecule of this ideal gas. Now calculate the ratio of specific heats for O_2 considering it as an ideal gas. 3 + 1

17. (*a*) The equation of state of a non-ideal gas is given by : $P(V-b) = RTe^{-a/RTV}$. Show that if a and b are small, this gas becomes a van der Waals' gas. 2

(*b*) Calculate the value of the molecular diameter of oxygen molecule for which van der Waals constant b is equal to $3{\cdot}186 \times 10^{-5}$ m^3 mol^{-1}. Given that Avogadro number is 6.023×10^{23}. 2

18. Obtain the direction and magnitude of magnetic field at a point on the axis of a circular current loop of radius a having n number of turns and carrying current I Amp. 4

19. A sphere of radius R contains a charge $+Q$ distributed uniformly in the upper hemisphere, and a charge $-Q$ distributed uniformly in the lower hemisphere. Show that the dipole moment of the charge distribution is $\frac{3}{4}QR\hat{z}$ where $\hat{z}$ is the unit vector along the polar axis of the spherical coordinate system. 4

20. A uniformly charged sphere of radius R carries a total charge Q. Show that the electrostatic energy is

$$U = \frac{3Q^2}{20\pi \epsilon_0 R}.$$ 4
