2018

PHYSICS

[Honours]

PAPER –I

Full Marks : 90

Time : 4 hours

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

GROUP – A

Answer any **two** questions : 15 × 2

1. (*a*) Consider the following function

$$f(x) = -\pi \text{ for } -\pi < x < 0$$
$$= 0 \text{ for } 0 < x < \pi$$

(*i*) Plot the function $f(x)$ vs x

(*Turn Over*)

(*ii*) Find the Fourier series expansion for $f(x)$

(*iii*)Hence deduce that

$$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots\cdots=\frac{\pi^2}{8}.$$

(*b*) Laplace equation in spherical polar co-ordinate for a problem with azimuthal symmetry is given by

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right)=0$$

Let $V(r,\theta)=R(r)\Theta(\theta)$, taking separation constant to be $l(l+1)$, Solve for $R(r)$. Also show that substituting of $x=\cos\theta$ in the angular part leads to Legendre's equation for $\Theta(\theta)=P(x)$.

(*c*) Using the generating function

$$G(x,t)=\left(1-2xt+t^2\right)^{-\frac{1}{2}}=\sum_{n=0}^{\infty}t^nP_n(x)$$

where $|t|<1$, find out the Legendre polynomial $P_2(x)$. (1 + 4 + 2) + (3 + 3) + 2

2. (*a*) Find the directional derivative of $\phi = x^3 + y^3 + z^3$ at the point (1, −1, 2) in the direction of the vector $\hat{i} + 2\hat{j} + \hat{k}$.

(*b*) State Gauss' divergence theorem. Verify divergence theorem for $\vec{F} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$ taken over the cube $0 \le x, y, z \le 1$.

(*c*) Expand $f(x) = \ln(1 + x)$ in a Taylor's series about the origin.

(*d*) Find a power series solution to the differential equation

$$\frac{d^2y}{dx^2} + y = 0$$

3 +(1 + 5) + 2+ 4

3. Consider a stretched uniform flexible string fixed at both ends.

(*i*) Obtain the differential equation for transverse vibrations of the string

(*ii*) Solve the equation to find the displacement for the *n-th mode.*

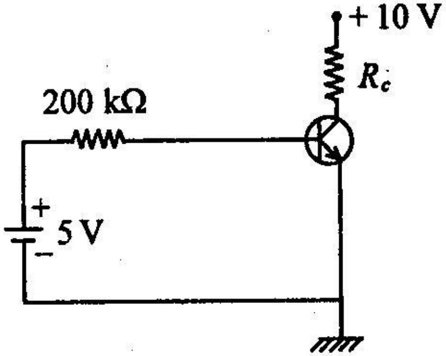
(*iii*) Obtain an expression for the frequency of the n-th mode.

(*iv*) The energy in the n-th mode is given by $E_n = \frac{1}{4} m A_n^2 W_n^2$, where m is the mas of the string, A_n is the amplitude and W_n is the circular frequency in the n-th mode. Express the above result in terms of the maximum velocity of an antinode. 4 + 5 + 3 + 3

4. (*a*) Using the paraxial approximation, construct the system matrix for a thin lens made with a material of refractive index n and radii of curvature r_1 and r_2 respectively.

(*b*) Using Huygens' principle show that $\frac{1}{v} + \frac{1}{u} = \frac{2}{r}$ for a concave spherical mirror of radius of curvature r. u and v are the object distance and image distance respectively.

(*c*) A silicon transistor with $V_{BE(sat)} = 0{\cdot}8$ V, $\beta = 100$ and $V_{CE(sat)} = 0{\cdot}2$ V is used in the circuit shown here. Find the minimum value of R_c for which transistor remains in saturation.



(*d*) Explain with diagram, how a zener diode can be used as a voltage regulator. 4 + 4 + 4 + 3

GROUP – B

Answer any **five** questions : 8 × 5

5. (*a*) Show that $\nabla r^n = nr^{n-2}\vec{r}$.

(*b*) If $\vec{v} = \vec{\omega} \times \vec{r}$ then prove that

$$\vec{\omega} = \frac{1}{2}\nabla \times \vec{v},$$

where $\vec{\omega}$ is a constant vector.

(*c*) Prove

$$nP_n(x) = (2n-1)xP_{n-1}(x) - (n-1)P_{n-2}(x)$$

2 + 3 + 3

6. (*a*) A beam of uniform cross section clamped at one end and loaded at the free end. Find the frequency of small oscillation of such light beam.

(*b*) Deduce an expression for the couple required to twist a uniform solid cylinder by an angle.

(*c*) Prove that if a number of rods of torsional rigidities C_1, C_2, C_3 ... etc. are joined end to end, the torsional rigidity of the combination is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$

3 + 3 + 2

7. (*a*) State and explain Fermat's principle. From this principle, derive Snell's law of refraction at a spherical surface.

(*b*) Determine the matrix which will represent the effect of refraction through a spherical surface of radius of curvature R. (1 + 3) + 4

8. (*a*) Distinguish between Fresnel and Fraunhofer diffraction.

(*b*) Outline the theory of zone plate and compare it with a converging lens. 2 + (4 + 2)

9. (*a*) Give a circuit diagram of an inverter with positive logic using an *n-p-n* transistor. Explain its operation.

(*b*) Simplify the Boolean expressions :

(*i*) $Y = (A+B)(A+\overline{B})(\overline{A}+B)$

(*ii*) $Y = (\overline{A+B})(\overline{A}+\overline{C})(\overline{B}+\overline{C})$

(*c*) Given the logic equation $Y = A\overline{B} + \overline{A}B$, draw its circuit diagram using basic gates.
(1 + 2) + (2 + 2) + 1

10. (*a*) Discuss analytically and graphically the variation of particle displacement and acoustic pressure in a plane progressive wave.

(*b*) Derive a relationship between group velocity and phase velocity.

(*c*) For gravity waves in liquid, the phase velocity is given by $v_p = \sqrt{\frac{\lambda g}{2\pi}}$, where λ is the wave-length. Find the group velocity. 3 + 3 + 2

11. (*a*) A diffraction grating used at normal incidence gives a line 5400 Å in certain order superposed on another line 4050 Å of the next higher order. If the angle of diffraction be 30°, how many lines/cm are there on the grating.

(*b*) Calculate the least width a grating must have to resolve the D-lines of sodium (589.0 nm and 589.6 nm) in the second order. The number of lines/mm of the grating is 300.

(*c*) State and explain Rayleigh's criterion of resolution. 3 + 3 + 2

12. (*a*) A series LCR circuit with $L = 0.05$ H,

$C = 50\ \mu F$ and $R = 10\ \Omega$ is connected to an alternating supply at 200 V and 50 Hz. Find the

(*i*) peak value of the current in the circuit

(*ii*) Power factor of the circuit

(*iii*) average power delivered to the circuit in each cycle.

(*iv*) rate of production of heat in the circuit.

(*b*) A 6·8 V, 300 mw zener diode is used as a voltage regulator with load resistance $R_L = 1\ k\Omega$ and a series resistance $R_S = 220\ \Omega$. Find the minimum and maximum values of input voltage for which output will be maintained constant at 6·8 V. 4 + 4

GROUP – C

Answer any five questions : 4 × 5

13. Expand $f(x) = \cos x$ as a Taylor series about $x = \frac{\pi}{3}$. 4

(Turn Over)

14. What is an emitter follower ? What are its inportant characteristics ? Draw the circuits diagram of an emitter follower and mention one use of it. 4

15. Consider a force field

$$\vec{F} = \frac{-y\hat{i} + x\hat{j}}{x^2 + y^2}.$$

Find $\oint \vec{F} \cdot d\vec{r}$ around a circular path of radius R with origin at the centre. Is $\vec{F}$ conservative. 4

16. (*a*) Why light emitted by two candles cannot produce an interference pattern ?

(*b*) Can we get interference with wide slits ? 2 + 2

17. What is meant by diffraction of light ? How is it different from interference ? 4

18. (*a*) Distinguish between ordinary and extra -ordinary rays.

(*b*) Find the state of polarisation when x and y component of the electric field are given by

$$E_x = E_0 \sin(wt + kz)$$

$$E_y = E_0 \cos(wt + kz)$$ 2 + 2

19. (*a*) Convert $(13{\cdot}65625)_{10}$ to a binary number

(*b*) IF $(2478)_{10} = (X)_{16}$ find X. 2 + 2

20. Show that for a forced vibration, the total energy of the vibrating system is not constant and in such case,

$$\frac{\text{Average Potential Energy}}{\text{Average Kinetic Energy}} = \frac{W_0^2}{W^2}$$

W_0 = natural angular frequency = $\sqrt{\frac{s}{m}}$ 4
