 UG/III/MATH/H/VIII/18(New)

2018

MATHEMATICS

[Honours]

PAPER – VIII

Full Marks : 60

Time : 3 hours

The figures in the right hand margin indicate marks

[NEW SYLLABUS]

GROUP—A

(*Numerical Analysis*)

[*Marks* : 25]

1. Answer any *two* questions : 8×2

(*a*) Derive the Newton Cote's formula (closed type) for n sub-intervals to determine the value of $\int_a^b f(x)dx$. Hence deduce the formula for $n = 2$. Find the degree of the polynomial for which Simpson's 1/3 rule gives exact value. 4 + 2 + 2

(*b*) (*i*) Define divided difference upto third order. Show that these differences are independent of the order of the arguments in which they appear. 4

(*ii*) Describe the Regula-Falsi method for computing a simple real root of an equation $f(x) = 0$ and give its geometrical interpretation. 4

(*c*) (*i*) A function $f(x)$ defined in $[0, 1]$ in such a way that $f(0) = 0$, $f\left(\frac{1}{2}\right) = -1$, and $f(1) = 0$. Find the interpolating polynomial $p(x)$ approximating $f(x)$. If $\left|\frac{d^3 f}{dx^3}\right| \le 1$ for $0 \le x \le 1$, show that

$$|f(x) - p(x)| \le \frac{1}{2}.$$

2 + 2

(*ii*) What types of methods are available in numerical analysis to get the solution of a system of linear algebraic equations ? State the condition for convergence of Gauss-Seidal method for numerical solution of a system of linear equations. 4

2. Answer any *three* questions : 3×3

(*a*) Explain loss of significant digit in numerical computation and give an example where this relation holds $(a + b) + c \neq a + (b + c)$ in numerical computations. 2 + 1

(*b*) Deduce the formula

$$\frac{d^r}{dx^r} f[x, x, \ldots r \text{ times } x] = r f[x, x, \ldots (r+1) \text{times } x]$$

and $\frac{d}{dx} f[x] = r!\, f[x, x, \ldots (r+1) \text{times } x]$,

where the symbols have their usual meanings. 3

(*c*) Prove that the Lagrange's formulae can be put in the form $P_n(x) = \sum_{r=1}^{n} \frac{\phi(x) f(x_r)}{(x - x_r)\phi'(x_r)}$

where $\phi(x) = \prod_{r=0}^{n} (x - x_r)$. 3

(*d*) Find the condition of convergence of the method of fixed point iteration for the numerical solution of an equation $f(x) = 0$. 3

(*e*) Write down the quadratic polynomial which takes the same value as $f(x)$ at $x = -1, 0, 1$ and integrate it to obtain the integration rule

$$\int_{-1}^{1} f(x)dx \cong \frac{1}{3} f(-1) + 4f(0) + f(1)$$ 3

GROUP—B

(*Real Analysis* - III)

[*Marks* : 25]

3. Answer any *one* question : 15×1

(*a*) (*i*) If a sequence of functions $\{f_n\}_n$ converges uniformly on $[a, b]$ to a function f and if $c \in [a, b]$ such that

$\lim_{x \to c} f_n(x) = a_n \quad (n \in N)$ show that

(*I*) $\{a_n\}_n$ converges

(*II*) $\lim_{x \to c} f(x)$ exists

and (*III*) $\lim_{x \to c} \lim_{n \to \infty} f_n(x) = \lim_{n \to \infty} \lim_{x \to c} f_n(x)$

2 + 2 + 2

(*ii*) Let $\{f_n(x)\} = \dfrac{a^2}{a^2 + n^2x^2}, a \neq 0.$

Show that $\{f_n(x)\}$ is uniformly convergent in $[A, B]$ where $0 < A < B$ but not uniformly convergent in $[-1, 1]$. 3

(*iii*) Let $I = [a, b]$ be a closed bounded interval and for each $n \in N, f_n : I \to \mathbb{R}$ be integrable on I to the function g then, prove that g is integrable on I and

$$\sum \int_a^b f_n(x)dx = \int_a^b g(x)dx.$$ 6

(*b*) (*i*) State and prove the Weierstrass M-test in connection with the uniform convergence of an infinite series of real-valued functions. Use this test to prove that the series $\sum_{n=1}^{\infty} \frac{(n+1)^3}{3^n \cdot n^5} x^n$ is uniformly convergent on $[-3, 3]$. $1+3+3$

(*ii*) Show that the series of functions $\sum_{n=1}^{\infty} \frac{\cos nx}{n^2}$ is uniformly convergent for real values of x. 5

(*iii*) State Dirichlet's condition for Fourier series. 3

4. Answer any *one* question : 8×1

(*a*) (*i*) Find the Fourier series of the periodic function $f(x)$ with period 2π, defined as :

$$f(x) = \begin{cases} -2 & \text{for } -\pi \le x < 0 \\ 2 & \text{for } 0 \le x < \pi \end{cases}$$ 4

(*ii*) Find radius of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{n!(x+2)^n}{n^n}.$$ 4

(*b*) (*i*) Show that

$$\sum_{n=1}^{\infty} \frac{x}{\{1+(n-1)x\}(1+nx)}$$

is uniformly convergent on any finite interval $[a, b]$, where $0 < a < b$. 4

(*ii*) If f is bounded and integrable on $[-\pi, \pi]$ and if

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx dx, (n = 0, 1, 2, ...)$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx, (n = 0, 1, 2, ...)$$

Then prove that the series

$$\sum_{n=1}^{\infty}(a_n^2 + b_n^2) \text{ converges.}$$ 4

5. Answer any *one* question : 2×1

(*a*) If $f(x)$ be the sum function of the power series $\sum_{n=0}^{\infty} a_n x^n$ on $(-R, R)$ for some $R > 0$ and if $f(x) = -f(-x)$ for all $X \in (-R, R)$, prove that $x_n = 0$ for all even n. 2

(*b*) Show that if a sequence of functions $\{f_n(x)\}$ is uniformly convergent on $[a, b]$ then it is pointwise convergence on $[a, b]$. 2

GROUP–C

(*Linear Algebra*)

[*Marks* : 10]

6. Answer any *one* question : 8×1

(*a*) (*i*) Prove that a linear transformation $L : V \rightarrow W$ is non-singular if and only if the set $\{Lx_1, Lx_2 \ldots Lx_n\}$ is a basis of W whenever the set $\{x_1, x_2 \ldots x_n\}$ is a basis of V. 5

(*ii*) The matrix $m(\mathring{T})$ of a linear mapping $T:\mathbb{R}^3 \to \mathbb{R}^2$ relative to the ordered bases $((0, 1, 1), (1, 0, 1), (1, 1, 0))$ of $\mathbb{R}^3$ and $((1, 0), (1, 1))$ of $\mathbb{R}^2$ is $\begin{pmatrix} 1 & 2 & 4 \\ 2 & 1 & 0 \end{pmatrix}$. Find T. 3

(*b*) When a linear transformation is said to be invertible ? If a linear transformation is invertible then prove that the inverse transformation is also linear. Let a linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ is defined by $T(x, y, z) = (x - y, x + 2y, y + 3z)$, $(x, y, z) \in \mathbb{R}^3$. Show that T is invertible and determine its inverse T^{-1}. $1+3+4$

7. Answer any *one* question : 2×1

(*a*) Let V and W be vector spaces over a field F. Let $T: V \to W$ be a linear mapping. Then prove that Ker T is a subspace of V. 2

(*b*) Define Nullity and Rank of a linear mapping. Give example. 2
