 UG/II/MATH/H/III/18(New)

2018

MATHEMATICS

[Honours]

PAPER – III

Full Marks : 90

Time : 4 hours

The figures in the right-hand margin indicate marks

[NEW SYLLABUS]

GROUP – A

(*Vector Analysis*)

[*Marks* : 25]

1. Answer any *one* question : 8 × 1

(*a*) (*i*) If $\frac{d\vec{f}(t)}{dt}$ exists at $t = t_0$, then prove that $\vec{f}(t)$ is continuous at $t = t_0$. Is the

(*Turn Over*)

converse true ? Illustrate your answer by an example. 4

(*ii*) Prove that for any two vector functions $\vec{f}$ and $\vec{g}$,

$$\operatorname{div}\left(\vec{f}\times\vec{g}\right)=\vec{g}\cdot\operatorname{curl}\vec{f}-\vec{f}\cdot\operatorname{curl}\vec{g}.$$ 4

(*b*) (*i*) A particle moves along the curve

$$x = 2t^2, y = t^2 - 4t, z = 3t - 5$$

where t is the time. Find the component of its velocity and acceleration at time $t = 1$ in the direction $\vec{i} - 3\vec{j} + 2\vec{k}$. 4

(*ii*) Show that the line integral

$$\int_P^Q x^2y\,dx + xyz\,dy + y^3dz$$

is not independent of the path of integration, where P and Q are respectively (0, 0, 0) and (1, 1, 1). 4

2. Answer any *three* questions : 4 × 3

(*a*) Verify Green's theorem in a plane for

$$\oint_C \left\{(x^2+xy)dx+xdy\right\}$$

where C is the curve enclosing the region bounded by $y=x^2$ and $y=x$. 4

(*b*) State and prove Gauss divergence theorem. 4

(*c*) Let $\vec{F}$ be a given vector field such that the domain D is either an open or a closed region and $\vec{F}$ posscesses continuous partial derivatives. Then prove that the necessary and sufficient condition for the field to be conservative if $\vec{F}$ is a gradient of some scalar field ϕ. 4

(*d*) Show that the acceleration $\vec{a}$ of a particle which travels along a space curve with velocity v is given by

$$\vec{a}=\frac{dv}{dt}\hat{T}+\frac{v^2}{e}\hat{N},$$

where $\hat{T}$ and $\hat{N}$ are unit tangent and normal *vectors to the curve.* 4

(*e*) Using the divergence theorem convert the surface integral

$$\oint_S x^3(xdydz + ydzdx + zdxdy)$$

into a volume integral and then evaluate the integral, where S is the surface of the closed cylinder whose curved surface is $x^2 + y^2 = a^2$ and the base are $z = 0$ and $z = b$. 4

3. Answer any *one* question : 3 × 1

(*a*) Show that the integral

$$\oiint xdydz + ydzdx + zdxdy$$

over the surface of a sphere equals three times its volume.

(*b*) Find the derivative of the scalar field $u = x^2 - y^2$ at the point $(5, 4)$ of the hyperbola $x^2 - y^2 = 9$ in the direction of the curve.

4. Answer any *one* question : 2 × 1

(*a*) Find the angle between the gradients of the functions $u=|\vec{r}|$ and $v=\log|\vec{r}|$ at $P(0, 0, 1)$. 2

(*b*) Evaluate

$$\oint_{\Gamma}\left(e^{x}dx+2y\,dy-dz\right)$$

by using Stokes's theorem, where Γ is the curve $x^2+y^2=4,\ z=2$. 2

GROUP –B

(*Analytical Geometry of three Dimensions*)

[*Marks* : 30]

5. Answer any *one* question : 15 × 1

(*a*) (*i*) Find the equation of a cone whose vertex is the point $P(x, y, z)$ and whose generating lines pass through the conic

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ z=0.$$

If the section of this cone by the plane

$z = 0$ is a rectangular hyperbola, show that the locus of P is

$$\frac{x^2}{a^2} + \frac{y^2 + z^2}{b^2} = 1.$$ 7

(*ii*) Show that the equation

$$2y^2 - 2yz + 2zx - 2xy - x - 2y + 3z - 2 = 0$$

represents a hyperbolic cylinder and find the equations of its axis. 8

(*b*) (*i*) A sphere of constant radius r passes through the origin O and cuts the axes in A, B, C. Prove that the locus of the foot of the perpendicular from O to the plane ABC is given by

$$\left(x^2 + y^2 + z^2\right)^2 \left(x^{-2} + y^{-2} + z^{-2}\right) = 4r^2.$$ 7

(*ii*) Find the equation of the ruled surface generated by a variable straight line which is always parallel to the plane $x = 0$ and intersects the line $x + z = 1,\ y = 0$ and the parabola $y^2 = 4x,\ z = 0$. 8

6. Answer any *one* question : 8 × 1

(*a*) A variable sphere passes through the points $(0, 0, \pm c)$ and cuts the straight lines $y = x\tan\alpha$, $z = c$ and $y = -x\tan\alpha$, $z = -c$ at the points P and P'. If $PP' = 2a$, (a is a constant), show that the centre of the sphere lies on the circle

$$x^2 + y^2 = (a^2 - c^2)\operatorname{cosec}^2 2\alpha,\ z = 0.$$

(*b*) Show that the perpendiculars from the origin to the generators

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 \text{ lie upon}$$

$$\frac{a^2(b^2 + c^2)^2}{x^2} + \frac{b^2(c^2 + a^2)^2}{y^2} = \frac{c^2(a^2 - b^2)^2}{z^2}.$$

7. Answer any *one* question : 4 × 1

(*a*) Prove that the enveloping cylinder of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1,$$

whose generators are parallel to the lines

$$\frac{x}{0} = \frac{y}{\pm\sqrt{a^2 - b^2}} = \frac{z}{c}$$

meet the plane $z = 0$ in circles.

(*b*) Find the equations to the generating lines of the hyperboloid

$$yz + 2zx + 3xy + 6 = 0$$

which pass through the point $(-1, 0, 3)$.

8. Answer any *one* question : 3 × 1

(*a*) If α, β, γ be the direction angles of a straight line, then show that

$$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$$

(*b*) Find the equations of the straight line that intersects the straight lines

$$x + y + z - 4 = 0 = 2x - y - z - 3,$$
$$x - y + z - 3 = 0 = x + 4y - z + 1$$

and passes through the point $(1, 0, 1)$.

GROUP – C

(Linear Programming and Game Theory)

[*Marks* : 35]

9. Answer any *one* question : 15 × 1

(*a*) (*i*) Find the dual of the following L.P.P and hence solve it. What is the advantages to use duality to solve an L.P.P.

Maximize $Z = 3x_1 - 2x_2$

subject to $x_1 \leq 4$

$x_2 \leq 6$

$x_1 + x_2 \leq 5$

$-x_2 \leq -1$

and $x_1, x_2 \geq 0$ 7

(*ii*) Prove that if for a BFS X_B of a LPP

Maximize $Z = cx$

Subject to $Ax = b, x \geq 0$

we have $z_j - c_j \geq 0$ for every column a_j of A, then X_B is an optimum solution. 8

(*b*) (*i*) Solve the following Travelling salesman problem. A salesman has to visit five cities. He wishes to start from a particular city, visit each city once and return to the starting city. The cost of going from a city to another in Rs. is given below. 7

From	To city					
		A	*B*	*C*	*D*	*E*
City ↓	*A*	–	12	15	17	11
	B	16	–	15	18	12
	C	18	17	–	17	17
	D	21	14	18	–	16
	E	11	13	12	18	–

(*ii*) If an LPP optimize $z = cx$ subject to $Ax = b, x \geq 0$, where A is the $m \times n$ coefficient matrix $(m < n)$ and $R(A) = m$ has an optimal solution, then there exists at least one BFS which will be optimal. Prove it. 8

10. Answer any *two* questions : 8 × 2

(*a*) Using Hungarian method, solve the following assignment problem for minimum total cost.

Jobs

Typist		P	Q	R	S	T
	A	85	78	65	90	60
	B	100	80	50	75	82
	C	75	76	40	62	66
	D	90	72	38	87	92
	E	86	80	85	72	82

Find the optimum assignments of jobs to typists and the minimum total cost. Is the solution obtained by you unique ? 8

(*b*) (*i*) State the rules of dominance for two persons zero-sum matrix games. 3

(*ii*) Use graphical method to solve the following game : 5

		B: I	B: II
A	I	2	–2
	II	–1	4
	III	5	–3
	IV	–2	1
	V	6	0

(*c*) Using dual simplex method, solve the following LPP :

Minimize $Z = 10x_1 + 6x_2 + 2x_3$

subject to $-x_1 + x_2 + x_3 \geq 1$

$3x_1 + x_2 - x_3 \geq 2$

$x_1, x_2, x_3 \geq 0.$ 8

11. Answer any *one* question : 4 × 1

(*a*) (*i*) Define convex polyhedron with example. 2

(*ii*) Distinguish between a regular simplex method and a dual simplex method. 2

(*b*) (*i*) Every transportation problem has an optimal solution. Justify it. 2

(*ii*) What do you mean by a standard form of an LPP ? 2