PG/IIS/MTM-201/15

M.Sc. 2nd Semester Examination, 2015

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(Fluid Mechanics)

PAPER – MTM- 201

Full Marks : 50

Time : 2 hours

Answer **Q.No.1** and any **three** questions from the rest

The figures in the right hand margin indicate marks

1. Answer any *two* questions : 2 × 2

(*a*) What is the kinematic coefficient of viscosity ?

(*b*) State Kelvin's circulation theorem.

(*c*) What do you mean by a vortex ?

2. (*a*) State and prove Blasius Theorem. 7

(*b*) An elliptic cylinder, the semi-axes of whose cross-sections are '*a*' and '*b*', is moving with velocity U parallel to the major axis of the cross-section, through an infinite liquid of density ρ which is at rest at infinite, the pressure there being Π. Prove that in order that the pressure may every where be positive $\rho U^2 < \frac{2a^2\Pi}{2ab+b^2}$. 5

3. (*a*) A sphere of centre O and radius 'a' moves through an infinite liquid of constant density ρ at rest at infinity, O describing a straight line with velocity $V(t)$. If there in no body forces, show that the pressure p at points on the surface of the sphere in a plane perpendicular to the straight line at a distance x from O measured positively in the direction of V is given by

$$p = p_0 - \frac{5}{8}\rho V^2 + \frac{9}{8}\rho V^2\left(\frac{x}{a}\right)^2 + \frac{\rho}{2}x\frac{dV}{dt}.$$ 7

(*b*) Derive the velocity potential and stream function at any point of a liquid contained between coaxial cylinders of radii '*a*' and '*b*' ($a < b$) when the cylinders are moved parallel to themselves in directions at right angles with velocities U and V respectively. 5

4. (*a*) Find the expression for velocity of flow of viscous fluid flowing along the axis of a pipe of rectangular cross-section with no slip condition at walls. 6

(*b*) An infinite row of equivalent rectilinear vortices are at a distance '*a*' apart. The vortices are the same numerical strength K but they are alternately of opposite signs. Find the complex function that determines the velocity potential and the stream function. Show also that if α be the radius of the vortex, the amount of the flow between two vortex and the next is

$$\frac{K}{\pi}\log\left\{\cot\left(\frac{\pi\alpha}{2a}\right)\right\}.$$ 6

5. (*a*) Deduce Navier-Stokes Equations of motion. What is the difficulty to solve this equation ? 6 + 2

(*b*) Discuss about the dissipation of energy due to viscosity. 4

6. (*a*) A circular cylinder is moving in a liquid at rest at infinity. If U and V are velocity components along $x-$ and $y-$ directions then find the expression for resultant velocity and the pressure exerted by the fluid on it. 6

(*b*) Determine the expression for tangential stress at any point and drag per unit area when a viscous fluid is flowing through a parallel plate channel having lower plate is fixed and the upper plate is moving with a velocity V along the axis of the channel. The distance between the plate being d. 6

[*Internal Assessment* : 10 Marks]
