2015

M.Sc.

1st Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-105

Full Marks : 50

Time : 2 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

(*Classical Mechanics and Non-linear Dynamics*)

Answer *Q. No. 1* and any *four* questions from the rest.

1. Answer any *four* questions : 4×2

   (a) Prove that the gain of Kinetic energy of a moving particle is equal to the work done by the particle.

   (b) Explain holonomic and non-holonomic systems in a dynamical problem.

   (c) What do you mean by cyclic coordinate? Define Routhian of a dynamical system.

(Turn Over)

(d) Explain Canonical transformation.

(e) State Hamilton principle.

(f) If H is the Hamiltonian and f is any function depending on position, momenta and time, show that

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + [f, H]$$

2. Show that with respect to a uniformly rotating reference frame, Newton's second law for a particle of mass m acted upon by real force $\overline{F}$ can be expressed as :

$$\overline{F}_{eff} = \overline{F} - 2m\overline{\omega} \times \overline{V}_{rot} - m\overline{\omega} \times (\overline{\omega} \times \overline{r}).$$

Assume that the origins of the inertial and non-inertial coordinate systems are coincident where $\overline{F}_{eff}$ and $\overline{V}_{rot}$ represent respectively the effective force and velocity in rotating frame. 8

3. Let m_0 be the mass of a particle at rest and m be the mass of the same particle when it is moving with velocity v. Then show that in relativistic mechanics

$m = \dfrac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$, where c is the velocity of light. 8

4. Deduce Lagrange's equations of motion in case of connected holomonic system. 8

5. A body moves about a point 0 under no forces, the principle moments of inertia at 0 being 3A, 5A and 6A. Initially, the angular velocity has components $w_1 = n$, $w_2 = 0$, $w_3 = n$ about the corresponding principal axes. Show that at any time t,

$$w_2 = \frac{3n}{\sqrt{5}} \tan h\left(\frac{nt}{\sqrt{5}}\right)$$

and that the body ultimately rotates about the mean axis.

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6. Prove that :

$$J = \int_{x_0}^{x_1} F\left(y_1, y_2, \ldots, y_n, y'_1, y'_2, \ldots, y'_n, x\right) dx$$

will be stationary if y_1, y_2, ..., y_n are obtained by solving the following equations :

$$\frac{d}{dx}\left(\frac{\partial F}{\partial y'_j}\right) - \frac{\partial F}{\partial y_j} = 0, \ j = 1, 2, \ldots, n$$

where $y'_j = \frac{\partial y_j}{\partial x}$.

8

7. (a) If [X, Y] denotes the Poission bracket, then prove the following results :

(i) [X+Y, Z] = [X, Z] + [Y, Z]

(ii) if $q = \sqrt{2P}\sin Q$, $p = \sqrt{2P}\cos Q$, then $[Q, P] = 1$. 2×2

(b) If $2T = \dot{\theta}^2 + \theta^2\dot{\phi}^2$ and $V = \frac{1}{2}n^2\theta^2$, prove that the Hamilton's equations are :

$$\theta^2 = a^2\cos^2(nt+\alpha) + b^2\sin^2(nt+\alpha)$$

and $\tan(\phi+\beta) = \frac{b}{a}\tan(nt+\alpha)$,

where a, b, α, β are constants. 4

(Internal Assessment — 10 Marks)